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# Shielding properties of the uranium-iron system

Mohamed El-Sayed Nagy  
Iowa State University

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SHIELDING PROPERTIES OF THE URANIUM-IRON SYSTEM

by

Mohamed El-Sayed Soliman Nagy

A Thesis Submitted to the  
Graduate Faculty in Partial Fulfillment of  
The Requirements for the Degree of  
MASTER OF SCIENCE

Major Subject: Nuclear Engineering

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Signatures have been redacted for privacy

Iowa State University  
Of Science and Technology  
Ames, Iowa

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## INTRODUCTION

After the discovery of X rays, it was recognized that nuclear radiation, resulting from radioactive decay in material, could produce harmful effects on the living organism. A number of cases of injury, ranging from minor skin lesion to bone sarcoma and untimely death, were reported among radiologists and others who were exposed to excessive amounts of nuclear radiation. It was not until the early 1920's, however, that organized efforts were made to recommend safety measures to be used in the manipulation of radium. About ten years later, maximum permissible levels of exposure to radiation were proposed, and their general acceptance led to a marked decrease in the incidence of radiation injury.

The building of nuclear reactors resulted in a great expansion of the field of health physics, the basic purpose being to derive methods for protection from unwarranted exposures to nuclear radiations. The efforts of health physicists have been so successful that injuries due to radiation have been rare, in spite of the very high levels of activity associated with reactor operation, the treatment of spent fuel, and the handling of fission products.

Shielding materials must not only serve as an adequate shield but must also be available at a reasonable cost. Materials used in shielding differ according to the type of radiations to be shielded. In general shielding, materials may be divided into three broad categories, according to their function: (1) heavy or moderately heavy elements to attenuate the gamma radiation and to slow down very fast neutrons by inelastic collisions; (2) hydrogenous substances to moderate neutrons

by elastic collision, and (3) materials, notably those containing boron, which capture neutrons without producing high energy gamma rays. No single material can act as a satisfactory shield for all of the various forms of radiations. The most important shielding materials in use today include water, concrete, iron and lead.

In a nuclear reactor and its surrounding shield there are present, almost all known types of nuclear radiation. Neutrons and gamma rays are emitted both in the primary fission process and in some of the secondary processes. The fission fragments are radioactive and emit gamma rays, beta rays and neutrons. In addition, beta rays can result from various secondary processes caused by gamma rays and neutrons. Finally, as a result of nuclear reactions in the reactor core or shield, heavy charged particles such as protons, deuterons, alpha particles, etc. will appear.

Not all the radiation involved in a nuclear reactor are of equal importance for shielding. For example, the charged particles, by virtue of their electric charge, interact strongly with the atomic electrons of the matter, and very quickly lose their energy. They can thus be stopped in relatively thin layers of material. The fission fragments are usually stopped well within the fuel elements themselves. For other charged heavy particles (alpha particles or protons) the range will be longer, but still much less than the thickness of structures normally found in the reactor core. For beta particles with certain energies the range is much longer than for the heavy particles with the same energies. An electron of about 5-Mev energy has a range in air of 22 meters, but only 2-6 cm in water or 0.33 cm in lead. It is clear

that it is easy to stop electrons with the energies normally encountered. However, one must still be concerned with bremsstrahlung radiation which results due to the stopping of electrons with energies of several Mev by materials of high atomic number such as lead.

In handling radioactive wastes, spent reactor fuel elements or certain radioactive isotopes, gamma radiation constitutes the primary hazard. Concerning attenuation of gamma rays, the origin of the radiation is not of significance, it is only the energy which determines the attenuation in a given medium. The linear attenuation coefficient generally increases with the density for a given energy. This coefficient is very roughly proportional to the density. From the above discussion one can conclude that the thicknesses of different materials required to attenuate gamma radiation, of specified energy, to the same extent are inversely proportional to their respective densities. Consequently, where thickness of the shield is an important consideration, a material of high density would be needed to attenuate the gamma rays. Lead has been used as the major shielding material in these cases.

Lead has two main disadvantages as a shielding material: It has little structural strength and a relatively low melting point. Because of its low melting point, lead can be used only where encountered temperatures are not too high.

Depleted uranium obtained from spent fuel element or as a by-product of the gaseous diffusion operation can be used as a very efficient material for gamma ray attenuation. In the last few years it has been used in fuel transfer casks. Uranium has relatively poor mechanical properties. The pyrophoric property of uranium is one of the main draw-

backs of its use as a shielding material. Alloying of uranium can improve the mechanical properties as well as eliminate the pyrophoric property. Iron is an example of a cheap element with good mechanical and thermal properties that could be used as an alloying material. It is of importance, therefore, to investigate the gamma rays attenuation properties for the U-Fe system.

## THE URANIUM IRON SYSTEM

The uranium iron system was first investigated by Grogan (6). In this investigation, the iron used was metal of 99.8 per cent purity. The system was investigated over its entire composition range. Two inter-metallic compounds,  $U_6Fe$  and  $UFe_2$ , are formed in this system. Their melting points are  $805^\circ C$  and  $1230^\circ C$  respectively. The intermetallic compound  $U_6Fe$  forms an eutectic with  $UFe_2$  which contains about 11 wt per cent iron. The eutectic has a melting point of  $725^\circ C$ . The compound  $UFe_2$  forms an eutectic with iron which contains about 47 wt per cent iron. This eutectic has a melting point of  $1055^\circ C$ . The two inter-metallic compounds are found to be brittle, which in turn would cause machining difficulties (Blizard 13). Powder metallurgical means have been applied for  $UFe_2$ . Thus the brittle nature of the eutectic alloys  $U_6Fe$  and  $UFe_2$  prohibited conventional machining techniques. As a result, casting in a vacuum in the form of a cylinder was applied. By careful control of mold shape, cooling rate, etc., a U-Fe eutectic sample was prepared by McKee (8) with only a few visible radial cracks. Loch (7) showed that the crystal structure of  $U_6Fe$  is body centre tetragonal and it has a density of  $17.7 \text{ gm/cm}^3$ .  $UFe$  has a FCC structure and a density of  $13.21 \text{ gm/cm}^3$ .

Several uranium-iron alloys have been examined by the X ray diffraction method (Grogan 6). These alloys contained from 1 to 50 wt per cent iron. All the samples containing 4 per cent and more of iron were prepared by pounding in a steel pestle. Apart from impurities in the form of uranium oxide, only four phases were observed: the iron, uranium,  $UFe_2$  and  $U_6Fe$  phases.



There has not been much done about studies on the effect of irradiation on the dimensional stability of the different compositions. In general it is found (Gittus 3) that an increase in the iron content of the alloys increases the over-all contraction.

Buzzard (1) reported that  $UFe_2$  is stable up to the melting point  $1235^\circ C$ . As a shielding material the different compositions in the U-Fe system offer some definite advantage over either iron or uranium except for  $U_6Fe$  which is pyrophoric. It is the purpose of this thesis to determine just how effective alloys of different compositions in the U-Fe system would be as a shield against gamma radiation.

The compositions that were chosen for shielding investigation are  $U_6Fe$ , the eutectic ( $U_6Fe-UFe_2$ ),  $UFe_2$  and a spectrum in the region  $UFe_2$ -eutectic ( $U_6Fe-UFe_2$ ).

## ANALYSIS

Upon investigating the shielding properties of the different compositions in the system, an analytical expression will be derived which describes the attenuation of gamma radiation in an absorbing media. This expression will then be modified to furnish a particular solution which will predict the attenuation that can be expected in the different materials.

In order to express radiation dosage in a quantitative manner, it is necessary to have a suitable unit. Such a unit for radiation is the roentgen, represented by the symbol r, which uses the ionization of air as its basis. It is defined as "that quantity of x or gamma radiation such that the associated corpuscular emission per 0.001293 gm of air (corresponding to 1 cm<sup>3</sup> of dry air at 0°C and 760 mm Hg) produces, in air, ions carrying 1 esu of quantity of electricity of either sign" by Goldstein (5).

It may be pointed out that the radiation dose expressed in roentgens does not depend on the time during which it is received. The rate of absorption or dosage rate is stated in terms of roentgens per unit time, e.g., roentgens per hour or milliroentgens per hour (r/hr or mr/hr). The integrated product of the dosage rate and the exposure time gives the total dose received in roentgens. The appropriate expression for the dose rate can serve as a measure of the radiation intensity in a certain region. However, this is justifiable only for gamma rays of a specified energy. The radiation intensity is the rate at which the energy flows past a unit area at a given location, but the dosage rate in roentgens per unit time is a measure of the rate at which energy is absorbed in air at that point.

The amount of energy deposited in air by a roentgen of x or gamma rays can be calculated in the following manner. The unit electrical charge, i.e. the electronic charge, is  $4.8 \times 10^{-10}$  esu, and this is the quantity of electricity carried by each member (positive or negative) of an ion-pair. Consequently,  $1/4.8 \times 10^{-10}$ , i.e.  $2.08 \times 10^9$ , ion pairs are required to give a total charge of 1 esu of either sign. Hence, from the definition of the roentgen, the absorption of 1 r of gamma radiation in 0.00129 gm of air results in the formation of  $2.08 \times 10^9$  ion-pairs in air. The energy required to produce one ion-pair in air is known to be about 34.0 ev and so the energy required for the formation of  $2.08 \times 10^9$  ion-pairs is  $(34.0)(2.08 \times 10^9) = 7.08 \times 10^4$  Mev or 0.113 erg. Hence, this must be the energy deposited in 0.00129 gm of air by 1 r of radiation. The energy absorbed per gram of air per roentgen is thus  $0.113/0.00129 = 88$  ergs.

To get a relation between the flux and the dose rate, one assumes that at some particular point there is a flux  $\phi$  photons/cm<sup>2</sup> sec. If  $E_\gamma$  is the photon energy in Mev, the gamma energy flux is  $\phi E_\gamma$  Mev/cm<sup>2</sup> sec. If  $\mu_e$  cm<sup>-1</sup> is the energy absorption coefficient of air for the given gamma rays, the rate of energy absorption in the material is  $\phi E_\gamma \mu_e$  Mev/cm<sup>3</sup> sec. This energy absorption can be put in terms of roentgen. Since absorption of  $7.082 \times 10^4$  Mev/cm<sup>3</sup> of air is equivalent to absorption of one roentgen, then the photon flux in terms of roentgen per sec will be:

$$\frac{\phi \text{ photons}}{\text{cm}^2 \text{ sec}} = \frac{\phi \frac{\text{photons}}{\text{cm}^2 \text{ sec}} \times E \frac{\text{Mev}}{\text{photon}} \times \mu_e \text{ cm}^{-1}}{7.082 \times 10^4 \frac{\text{Mev}}{\text{cm}^3/\text{r}}}$$

$$= \frac{\phi E \mu_e}{7.82 \times 10^4} \frac{r}{\text{sec}} \quad (1)$$

$$1 \frac{r}{\text{sec}} = \frac{7.082 \times 10^4}{E \mu_e} \frac{\text{photons}}{\text{cm}^2 \text{ sec}} \quad (2)$$

In terms of milliroentgens per hour, which is a more convenient unit for radiation dose rates, one has

$$1 \text{ mr/hr} = \frac{1.96 \times 10^{-2}}{E \mu_e} \frac{\text{photons}}{\text{cm}^2 \text{ sec}} \quad (3)$$

This equation is the general expression to convert a photon flux of  $\phi$  photons/cm<sup>2</sup> sec to the corresponding dose rate in mr/hr.

### Properties of Gamma Rays

#### Sources of gamma rays

Gamma rays and X rays are two forms of electromagnetic radiation differing only in their origins. Gamma rays are produced in nuclear reactions, whereas X rays are caused by the excitation or removal of orbital electrons or by the deceleration of electrons.

These radiations consist of photons, each with energy  $E$  given by the expression

$$E = h\nu = \frac{hc}{\lambda} \quad (4)$$

where  $h$  = Planck's constant =  $6.55 \times 10^{-22}$  erg/sec

$c$  = velocity of light =  $3 \times 10^{10}$  cm/sec

$\nu, \lambda$  = frequency and wave length, respectively, associated with the wave nature of the radiation. The emission of gamma rays is a mechanism by which the energy of excitation of nucleus can be removed. Such excited states may accompany the decay of radio isotopes, or they may result from induced nuclear transmutations. The gamma rays accompanying a particular type of nuclear reaction are composed of photons with

either a single energy or a group of discrete energies. Typical energies of gamma rays range from a few Kev to several Mev. The interaction of gamma rays with matter is primarily by one of three mechanisms; namely, the photo electric effect, Compton scattering or pair production. There are a number of other mechanisms which also occur, but their effects are either small enough to be neglected or concern photon energies outside the range of reactor shielding interest. Hence, only the three primary processes will be considered here. In the photo electric effect a photon of energy  $h\nu$  interacts with the atom as a whole. Its energy is transferred to an electron, usually one in the innermost shell. The electron is ejected with a kinetic energy

$$E_{kin} = h\nu - E_b \quad (5)$$

where  $E_b$  is the binding energy of orbital electron. When the electron shell is refilled, one or more characteristic X rays with total energy  $E_b$  are emitted. In pair production the primary photon disappears, and its energy goes into the rest-mass energy and the kinetic energy of the positron and electron pair which is produced.

In both the photo electric and pair production interactions, the photon is essentially absorbed by the medium through which it passes. Thus, for a given narrow beam of photons, the number which succeed in passing through an absorbing medium will be just those which have not suffered a collision. The change in uncollided photon flux then will be proportional to the thickness of a thin absorber. Therefore, the flux change  $d\phi$  resulting from passage through a small thickness  $dx$  of matter, at any point in the medium, is proportional to the radiation intensity at that point and the thickness traversed, i.e.,

$$d\phi = \mu\phi dx \quad (6)$$

where  $\mu$  is the proportionality constant. It is the linear attenuation coefficient of the absorber for the given radiation. This coefficient is characteristic of the absorber, and is independent of the absorber thickness if the photon beam is monoenergetic. In this case the flux of the emergent beam at any point  $x$  is obtained by integrating Equation 6. The result is

$$\phi(x) = \phi_0 e^{-\mu x} \text{ photons/cm}^2 \text{ sec} \quad (7)$$

where  $\phi_0$  is the radiation flux before passage through a thickness  $x$  of shield.

The simple exponential attenuation Equation 7 is based on the tacit assumption that scattered particles are completely removed from the radiation beam. The quantity  $\phi(x)$  then gives what is called the uncollided flux, i.e., the radiation flux which has not been involved in any collision in its passage through a thickness  $t$  of the shield. In fact, with this definition of  $\phi(x)$ , Equation 7 is applicable to a broad collimated beam of radiation.

For a relatively thin layer of attenuating material, i.e., for a thin shield, Equation 7 is a good approximation for the measured flux, even for a broad collimated beam, especially for photons of high energy. This is because the probability that a scattered radiation particle will reach the observation point (or detector) after a single collision is small. The total flux measured is then essentially the same as the uncollided flux. On the other hand, if the shield is relatively thick, some particles which have suffered two or more scattering collisions within the absorber may reach the detector. In this case the scattered

particles are not removed, and the flux at the observation point exceeds the uncollided flux; the simple exponential equation will then give broad beam values for  $\phi(x)$  that are too low.

The effect of the scattered radiation can be accounted for by use of what is called the build-up factor. It is a function of the shield material and thickness as well as the energy of radiation and the particular quantity being observed. Thus, for a given shield and radiation, the value of the build-up factor would be different for the number flux, i.e., particles/cm<sup>2</sup> sec; for the energy flux, i.e., Mev/cm<sup>2</sup> sec; and for the dose rate, i.e., roentgens/sec, etc. Assuming the number flux is being observed, the attenuating Equation 7 would then be modified to take the form

$$\phi(x) = \frac{E \mu_e}{1.96 \times 10^{-2}} \phi_0 \beta(\mu x) e^{-\mu x} \quad \frac{mr}{hr} \quad (8)$$

where

$$\frac{E \mu_e}{1.967 \times 10^{-2}} = \text{ratio for converting flux in photons/cm}^2 \text{ sec to dose rate in mr/hr}$$

$\phi$  = radiation flux in photons/cm<sup>2</sup> sec before passage through a thickness  $x$  of absorber.

$\beta(\mu x) e^{-\mu x}$  = a dimensionless quantity which is that fraction of the original flux, both collided and uncollided, which succeeds in passing through the absorber.

Equation 8 is the general expression for the dose rate due to the fraction  $\beta(\mu x) e^{-\mu x}$  of the original gamma flux  $\phi_0$  that has passed through a thickness  $x$  of an absorbing medium. Although Equation 8 is completely general, care must be taken to account for the source geometry and the shielding material. In our specific case the shielding materials will be seven different compositions of the uranium-iron system. Those com-

positions are:  $U_6Fe$ , the eutectic compound ( $U_6Fe-UFe_2$ ), the intermetallic  $UFe_2$ , alloy with 35 wt per cent iron, alloy with 40 wt per cent iron, alloy with 45 wt per cent iron, and the eutectic compound ( $UFe_2-Fe$ ).

Assuming that one has an isotropic point source emitting  $S_p$  photons/sec, then the flux at a distance  $R$  cm, disregarding attenuation by the medium, will be  $S_p/4\pi R^2$  photons/cm<sup>2</sup> sec, since  $4\pi R^2$  is the area of a spherical surface of radius  $R$ . By combining this result with the equivalent of Equation 8, to allow for attenuation and build-up factor of the radiation by a shielding detector, it is found that the flux at the detector is given by

$$\phi = S_p \mu x \frac{e^{-\mu x}}{4\pi R^2} \quad (9)$$

The attenuation due to the thickness  $R-t$  of air is here neglected.

The source term  $S_p$  can be conveniently expressed in units of curies. A curie is defined as the activity of a certain amount of the given radioactive nuclide which undergoes  $3.7 \times 10^{10}$  disintegrations per second. Consider a radioactive point source having strength of  $c$  curies. Therefore

$$S_p = 3.7 \times 10^{10} \frac{\text{disintegrations}}{\text{sec curie}} \times c \text{ curies} \quad (10)$$

and the flux term  $\phi_0$  in Equation 9 becomes  $\phi =$

$$\frac{3.7 \times 10^{10} c \frac{\text{disintegrations}}{\text{sec}} \times 1 \frac{\text{photon}}{\text{disintegration}}}{4\pi R^2 \text{ cm}^2} \quad (11)$$

or  $\phi =$

$$\frac{3.7 \times 10^{10} c}{4\pi R^2} \text{ photons/cm}^2 \text{ sec} \quad (12)$$

It will be postulated that one gamma ray photon is produced in



each act of nuclear disintegration, so that the source yields  $3.7 \times 10^{10}$  photons per sec. Neglecting the attenuation of radiation by the air from the source to the point of observation, the gamma ray flux at the distance  $R$  from the unshielded source is  $3.7 \times 10^{10} / 4\pi R^2$  photons/cm<sup>2</sup> sec. This expression can be taken as  $\phi$  in Equation 13.

Upon combining this result with Equation 8 for the dose, one has

$$D = \frac{E \mu_e}{1.967 \times 10^{-2}} \times \frac{3.7 \times 10^{10}}{4\pi R^2} \times \beta(\mu_r) e^{-\mu_r r} \text{ mr/hr} \quad (13)$$

Equation 14 can be written as

$$D = 1.5 \times 10^{11} \frac{E \mu_e C}{R^2} \beta(\mu_r) e^{-\mu_r r} \text{ mr/hr} \quad (14)$$

in which  $R$  is the distance in cm from the source to the detector and  $r$  is the radius of the shield in cm.

The shielding properties of the different alloys and compounds can be studied by using the above equation. By solving the above equation, one can find the required thickness of composition to give a certain dose at the detector. From that thickness one then can calculate the weight to be needed for that specific dose rate. In order to find an appropriate solution to this equation, one must analyze the individual terms in the equation.

An examination of the variation of the energy absorption coefficient of air ( $\mu_e$ ) with energy has been discussed in Glasstone (4). It can be shown that  $\mu_e$  may be taken to have a constant value of  $2.5 \times 10^{-5} \text{ cm}^{-1}$  but since  $\mu_e$  actually varies with the photon energy, this result is accurate to within  $\pm 10$  per cent. If one uses this value for  $\mu_e$ , Equation 14 can be written as

$$D = 5.24 \times 10^6 \frac{R}{R^2} C \beta(\mu_r) e^{-\mu_r R} \text{ mr/hr} \quad (15)$$

Now consideration will be given to the calculation of the linear attenuation coefficients for the different compositions. According to Goldstein (5), if the attenuating medium consists of several elements forming a chemical compound or a homogeneous mixture, the mass absorption coefficient can be written as

$$\mu_m = \sum_i a_i \mu_i \text{ cm}^2/\text{gm} \quad (16)$$

where  $\mu_i$  is the mass absorption coefficient of the  $i^{\text{th}}$  element and  $a_i$  is the corresponding weight fraction.

Usually the absorption coefficient is expressed as a linear absorption coefficient, with units of reciprocal length. In metric units it can be expressed as follows

$$\mu(\text{cm}^{-1}) = \mu_m \text{ cm}^2/\text{gm} \rho \text{ gm/cm}^3 \quad (17)$$

where  $\rho$  is the density in  $\text{gm/cm}^3$ . Thus, the calculation of the linear absorption coefficient requires a knowledge of the density of the compound.

From the above discussion, the linear absorption coefficients of the different compositions of the U-Fe phase diagram, i.e.  $\text{U}_6\text{Fe}$ , the eutectic compound ( $\text{U}_6\text{Fe}-\text{UFe}_2$ ), the intermetallic  $\text{UFe}_2$ , alloy with 35 wt per cent Fe and 65 wt per cent U, alloy with 40 wt per cent Fe and 60 wt per cent U, alloy with 45 wt per cent Fe and 55 wt per cent U and the eutectic ( $\text{UFe}_2\text{-Fe}$ ) can be determined by knowing the atomic per cent of each individual constituent. Assuming a binary system AB, the mass absorption coefficient can be expressed as follows

$$\mu_m(\text{AB}) \text{ cm}^2/\text{gm} = \frac{\text{wt of A in gms/gm mole of AB}}{\text{molecular wt of AB in gms}} \times \mu_m(\text{A}) + \frac{\text{wt of B in gms/gm mole of AB}}{\text{molecular wt of AB in gms}} \times \mu_m(\text{B}) \quad (18)$$

From the Fe-U phase diagram; shown by Grogan (6), McKee(8) and Buzzard (1), one can note the intermetallic compounds, their densities and melting points as shown in Table 1.

One can calculate the density of the eutectic composition ( $\text{UFe}_2\text{-Fe}$ ) by using the following formula

$$D = 0.955 \times \left( \frac{100}{\frac{\text{wt per cent Fe}}{7.8} + \frac{\text{wt per cent U}}{19.05}} \right) \quad (19)$$

which is given by McKee (8). It was reported that this formula agrees with experimental results. The eutectic composition ( $\text{UFe}_2\text{-Fe}$ ) was reported by Grogan (6) to have 47 per cent iron. Therefore

$$D = 0.955 \times \left( \frac{100}{\frac{47}{7.8} + \frac{53}{19.05}} \right) = 10.82 \text{ gm/cc} \quad (20)$$

In the range ( $\text{UFe}_2$  --- ( $\text{UFe}_2\text{Fe}$ )) density of the alloys can also be calculated. The composition range is from (32→47) wt per cent iron. The compositions chosen are shown in Table 1 with their densities calculated according to Equation 19. The wt per cent of free iron and  $\text{UFe}_2$  in the different alloys could be calculated in the following way.

For an eutectic composition of 47 wt per cent Fe from the phase diagram, 100 gm of the eutectic contains 47 gm Fe. This means that there are 53 gm U and 47 gm Fe for every 100 gm eutectic. The molecular weight of  $\text{UFe}_2$  is  $(238.07 + 2 \times 55.85) = 349.67$ . Therefore, according to the eutectic composition, 238.07 gm of U have 111.6 gm Fe. Thus 53 gm U must have  $\frac{53 \times 111.6}{238.06}$  gm Fe or 24.8 gm Fe in  $\text{UFe}_2$ . The com-

position of 100 gm eutectic will be  $(53 + 24.8)$  gm  $UFe_2 = 77.8$  gm  $UFe_2$  and 22.2 gm Fe. Then at the eutectic composition there is 77.8 per cent  $UFe_2$  and 22.2 per cent Fe. Accordingly, compositions that were found in the other three alloys in the region ( $UFe_2 + Fe$ ) were calculated and summarized in Table 1.

The mass absorption coefficient of the different compositions can be calculated by using Equation 19 and Table 1 and by taking into consideration that the gm molecular wt of  $U_6Fe$ ,  $UFe_2$  and iron are 1484.27, 349.67 and 55.85 gm respectively. Table 2 shows the coefficient of the mass absorption coefficients of Fe and U to be used in Equation 18.

After calculating the mass absorption coefficients of the different compositions, one can calculate the linear absorption coefficients for each by multiplying by the density as was discussed earlier. Values of the linear and mass absorption coefficients are tabulated in Tables 1 and 2 respectively. The values of the mass absorption coefficients of iron and uranium are taken from LA-2237 (12).

According to Goldstein (5), the calculation of the build-up factor for a homogenous mixture can be determined by getting an equivalent atomic number for the mixture, even where the elements making up the mixture have highly separated atomic numbers. It is usually possible to find an equivalent single element having the same gamma-ray build-up properties as the mixture and the equivalent for the mixture can be found by comparing the shape of the linear absorption curve with the corresponding curves for individual elements, i.e. by plotting the ratio  $\mu/\mu_0$  v.  $E$ , until a reasonable match is found over the region of interest. This procedure was investigated by Moser (9).

Table 1. Weight per cents, densities and melting points of the compounds

Compound	U <sub>6</sub> Fe	UFe <sub>2</sub>	total	U	den-	Fe	°C mp
	wt pc	wt pc	Fe wt pc	wt pc	sity gm/cm <sup>3</sup>	wt pc-due to free Fe	
1) U <sub>6</sub> Fe	100	---	3.76	96.24	17.7	---	815
2) eutectic I (U <sub>6</sub> Fe-UFe <sub>2</sub> )	75	25	10.1	89.9	15.9	---	725
3) UFe <sub>2</sub>	---	100	32	68	13.2	---	1235
4) alloy I (UFe <sub>2</sub> + Fe)	---	94.1	35	65	12.1	5.9	alloy I (35 wt per cent Fe) -
5) alloy II (UFe <sub>2</sub> + Fe)	---	88.1	40	60	11.52	11.9	alloy II (40 per cent Fe) -
6) alloy III (UFe <sub>2</sub> + Fe)	---	81.8	45	55	11.05	18.2	alloy III (45 per cent Fe) -
7) eutectic II (UFe <sub>2</sub> - Fe)	---	77.8	47	53	10.82	22.2	1080

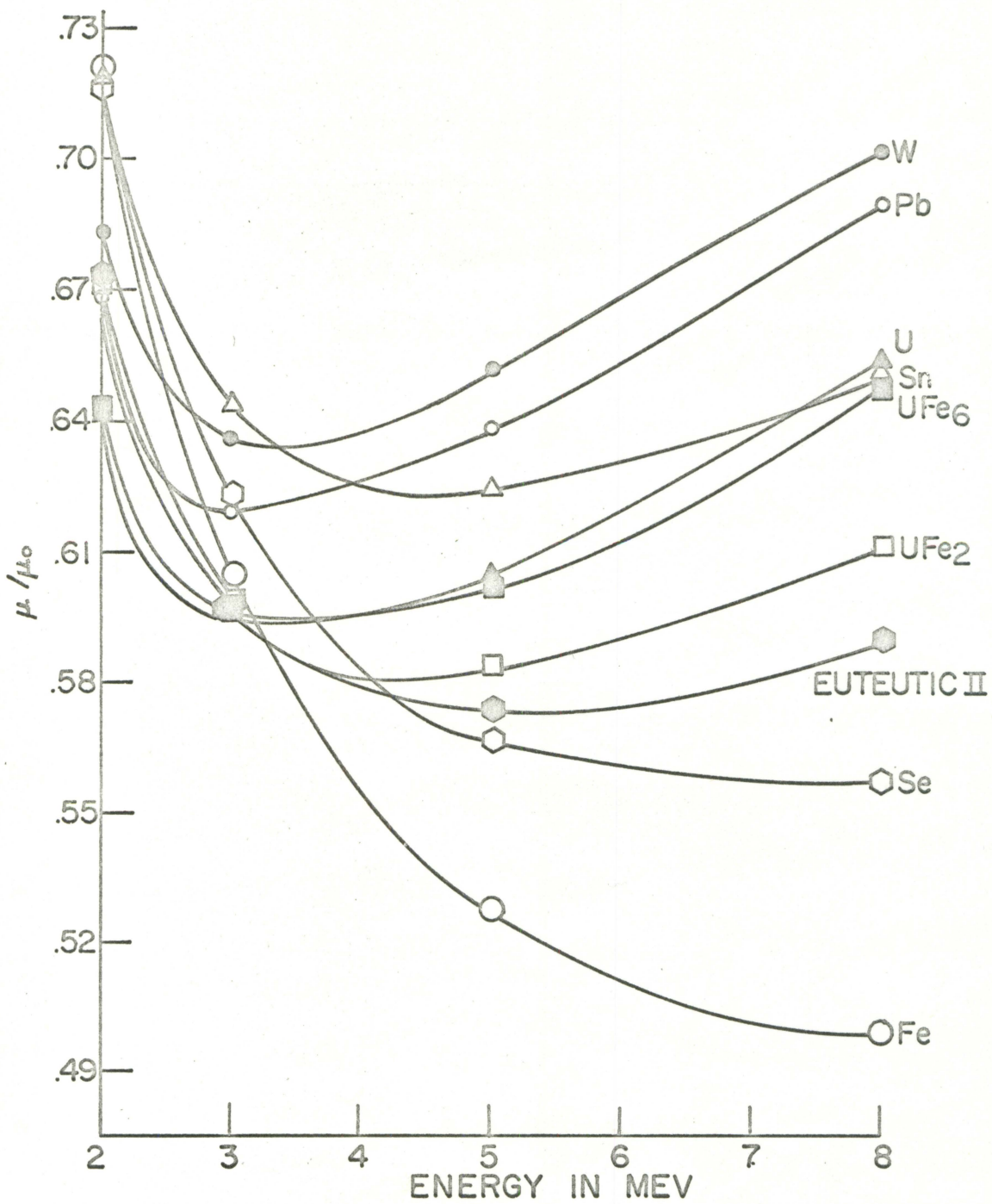
Table 2. Coefficients for calculating the mass absorption coefficients of the compounds

Compound	Coefficients for -	
	$\mu_m(\text{Fe})$	$\mu_m(\text{U})$
1) $\text{U}_6\text{Fe}$	0.0376	0.9624
2) eutectic I ( $\text{U}_6\text{Fe}-\text{UFe}_2$ )	0.1080	0.8920
3) $\text{UFe}_2$	0.3200	0.6800
4) alloy I (35 wt per cent Fe)	0.3600	0.6398
5) alloy II (40 wt per cent Fe)	0.4000	0.6000
6) alloy III (45 wt per cent Fe)	0.4440	0.5560
7) eutectic II ( $\text{UFe}_2-\text{Fe}$ )	0.4710	0.5290

This procedure for finding the build-up factor for the different compositions was tried but it failed to give any acceptable results as shown in Figure 1. The nature of this difficulty can be explained by studying the behavior of the attenuation coefficients of the different elements of atomic numbers between iron and uranium.

Since the preceding procedure for calculating the build-up factor failed, another approach for calculating the build-up factor was taken into consideration. According to Rockwell (11), the build-up factor of a homogeneous mixture of a binary system can be obtained by weighting the build-up factors of the elements according to the number of relaxation lengths of each of the individual elements present. Assume one has  $X$  mean free paths of a specific element where  $X = \mu \rho R$ .

Figure 1. Normalized mass absorption coefficients of Fe, Se, Eutectic II,  $UFe_2$ ,  $UFe_6$ , Sr, U and W





$\mu$  = mass absorption coefficient in  $\text{cm}^2/\text{gm}$

$\rho$  = density of the material in  $\text{gm}/\text{cm}^3$

$R$  = thickness in cm

If one has a thickness of  $R$  cm in the shield, then

$$(X_{\text{Fe}})_{\text{eq}} = \mu_{\text{Fe}} (\rho_{\text{Fe}})_{\text{eq}} R \quad (21)$$

$$(X_{\text{U}})_{\text{eq}} = \mu_{\text{U}} (\rho_{\text{U}})_{\text{eq}} R \quad (22)$$

The total mean free paths of the shield is then

$$X_{\text{T}} = (X_{\text{Fe}})_{\text{eq}} + (X_{\text{U}})_{\text{eq}}$$

$(X_i)_{\text{eq}}$  = equivalent mean free path if one considers the shield composed of element  $i$  only. The density of  $i$  is the equivalent density.

$(\rho_i)_{\text{eq}}$  = density of the  $i^{\text{th}}$  element in the alloy.

One can also define  $\beta_{\text{Fe}}(X_{\text{T}})$  and  $\beta_{\text{U}}(X_{\text{T}})$  as the build-up factor of the shield at  $X_{\text{T}}$ . By weighting the individual build-up factors by the equivalent mean free path length

$$\beta_{\text{T}} = (X_{\text{Fe}})_{\text{eq}} \beta_{\text{Fe}}(X_{\text{T}}) + (X_{\text{U}})_{\text{eq}} \beta_{\text{U}}(X_{\text{T}}) \quad (23)$$

The equivalent densities are tabulated in Table 3.

Using Table 3 and Equation 24 for the build-up factor, one can get the values of the build-up factors of the different compositions at different mean free path thicknesses.

The build-up factors for the different compounds and also for Fe, Pb and U that can be expected from gamma radiation of five different energies ranging from 1 to 8 Mev, are shown graphically in Figures 2 through 11. It can be seen that the value of the build-up factor is a function of the shield thickness, which is  $X_{(\text{r})}$ .

The general shape of the build-up factor as a function of thickness indicates that the build-up factors can be represented by a power series.

Table 3. Equivalent densities of the compounds

Compound	Compound gm/cm <sup>3</sup>	( $\rho_{Fe}$ ) <sub>eq</sub> gm/cm <sup>3</sup>	( $\rho_U$ ) <sub>eq</sub> gm/cm <sup>3</sup>
U <sub>6</sub> Fe	17.70	0.60	17.09
eutectic (U <sub>6</sub> Fe-UFe)	15.90	1.60	14.20
UFe <sub>2</sub>	13.20	4.20	8.90
alloy I	12.10	4.20	7.80
alloy II	11.52	4.60	6.90
alloy III	10.82	4.97	6.07
eutectic (UFe <sub>2</sub> -Fe)	10.82	5.08	5.73

With such series, the data for the build-up factor could be expanded into the form of a general equation which one can apply to find build-up factors for the intermediate gamma energies and shield thicknesses.

The equation has the form

$$\beta(\mu r) = 1 + \alpha_1(E)r + \alpha_2(E)r^2 + \alpha_3(E)r^3 + \dots \quad (24)$$

in which the coefficients are energy dependent. Equation 24 was applied to the calculated values of the build-up factors and appropriate values of three coefficients were found by applying a least square fit to the data. The values of the three coefficients have been tabulated in Table 5.

In an infinite medium, the build-up factor at some distance X would

Figure 2. Dose build-up factors for  $U_6Fe$  for gamma ray energies of 1.0, 2.0, 3.0, 5.0 and 8.0 Mev

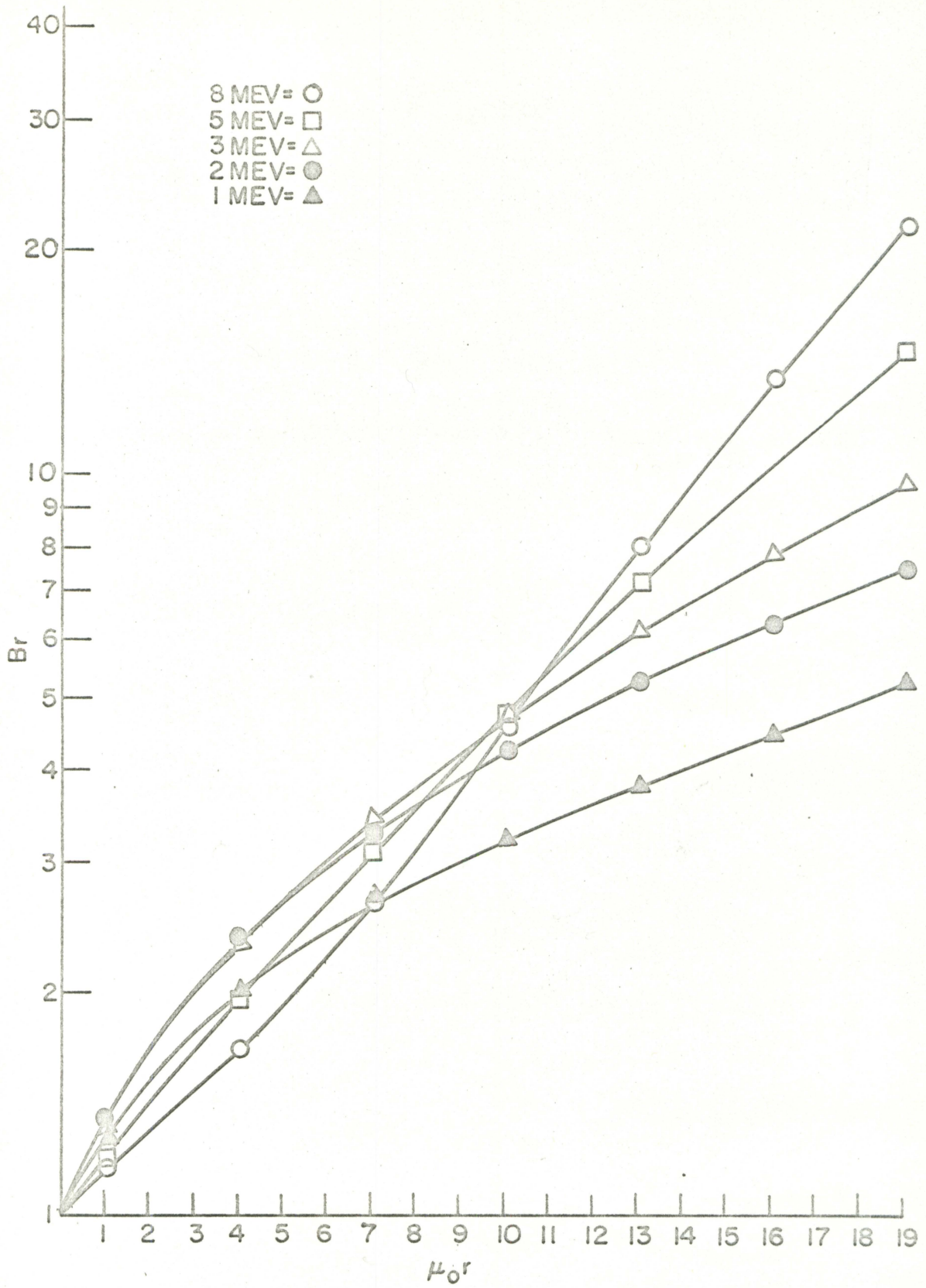


Figure 3. Dose build-up factors for eutectic ( $U_6Fe-UFe_2$ ) for gamma ray energies of 1.0, 2.0, 3.0, 5.0 and 8.0 Mev

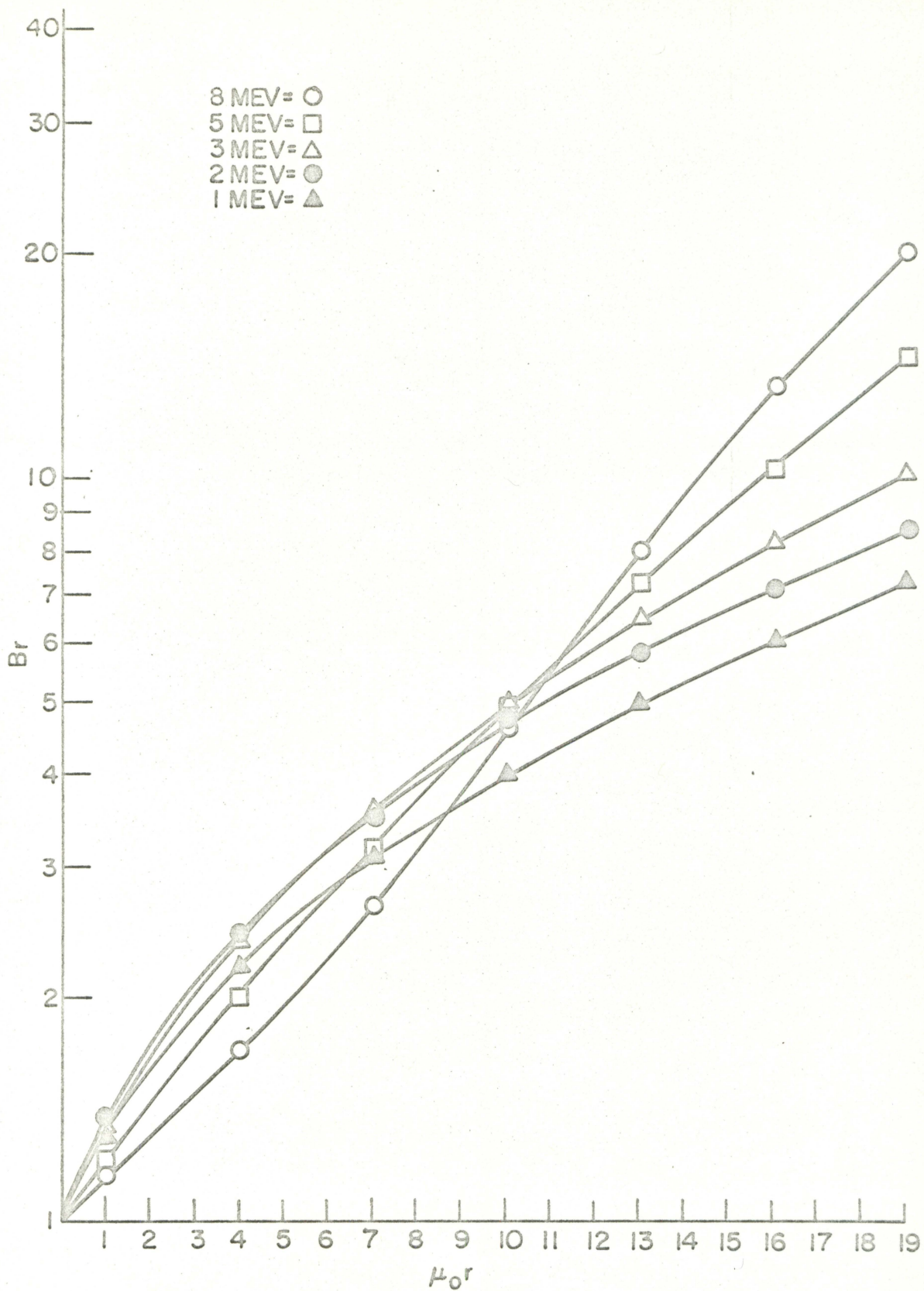


Figure 4. Dose build-up factors for ( $UFe_2$ ) for gamma ray energies of 1.0, 2.0, 3.0, 5.0 and 8.0 Mev<sup>2</sup>

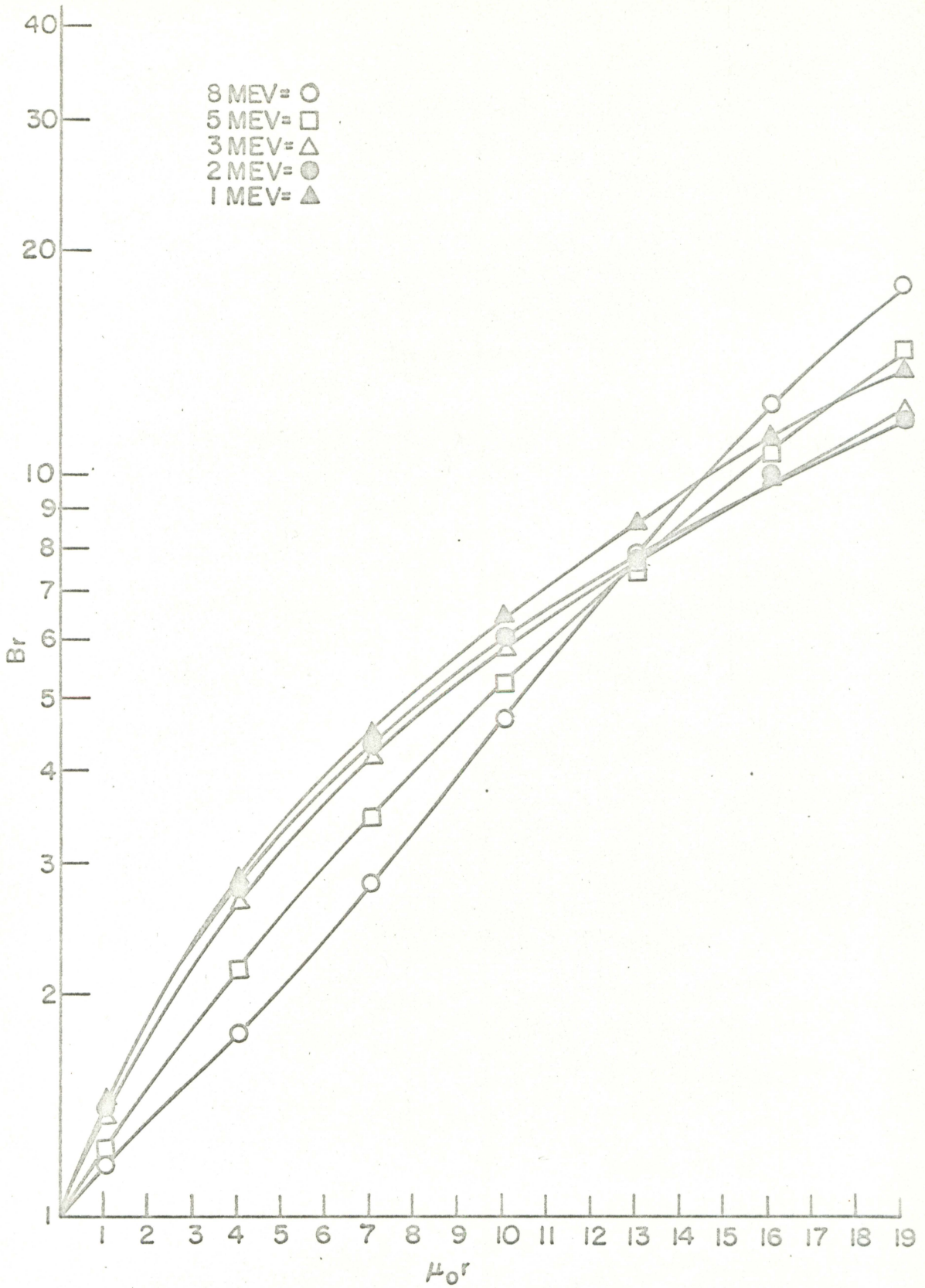




Figure 5. Dose build-up factors for alloy with 35 wt per cent Fe and 65 wt per cent U for gamma ray energies of 1.0, 2.0, 3.0, 5.0 and 8.0 Mev

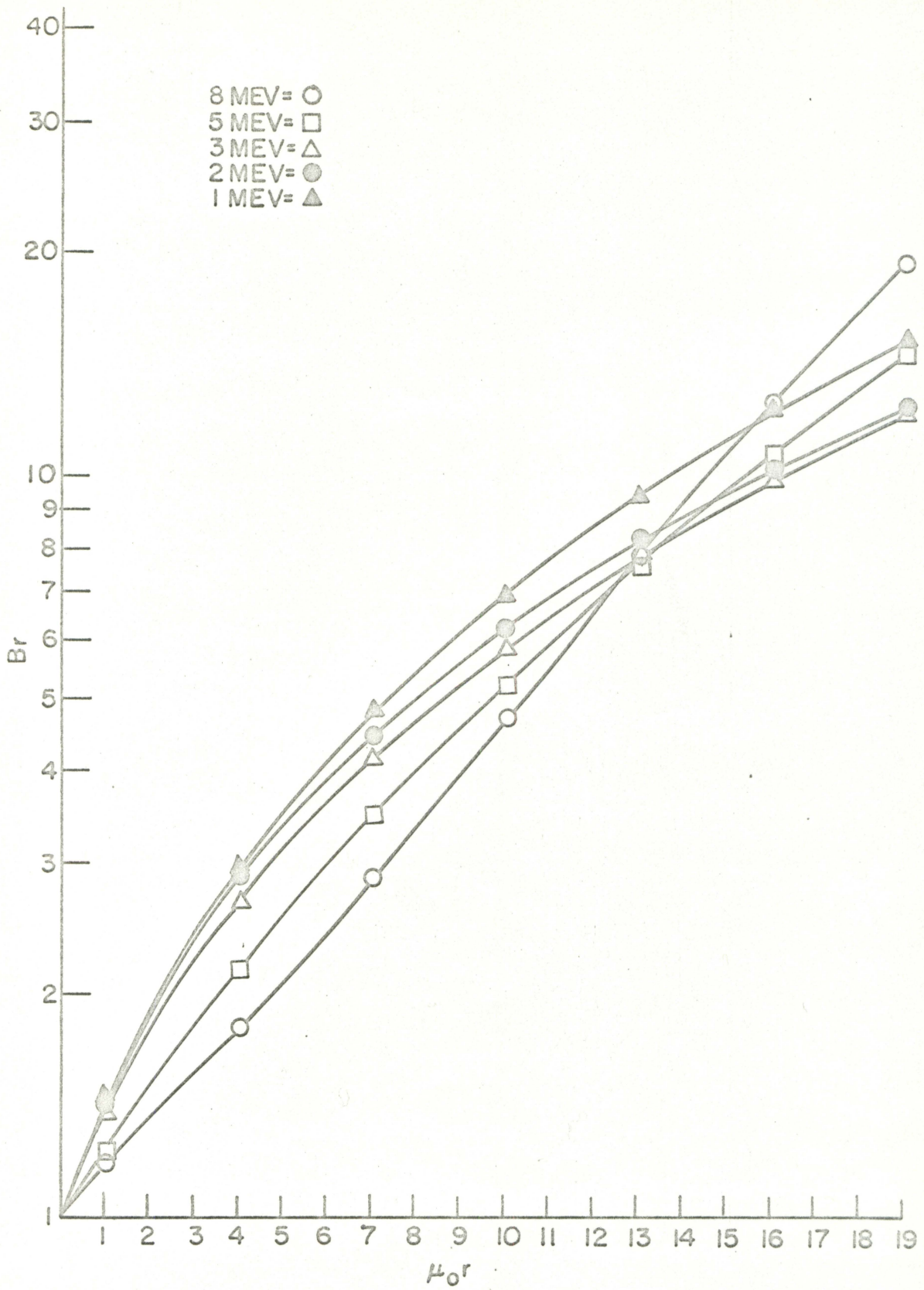


Figure 6. Dose build-up factors for alloy with 40 wt per cent Fe and 60 wt per cent U for gamma ray energies of 1.0, 2.0, 3.0, 5.0 and 8.0 Mev

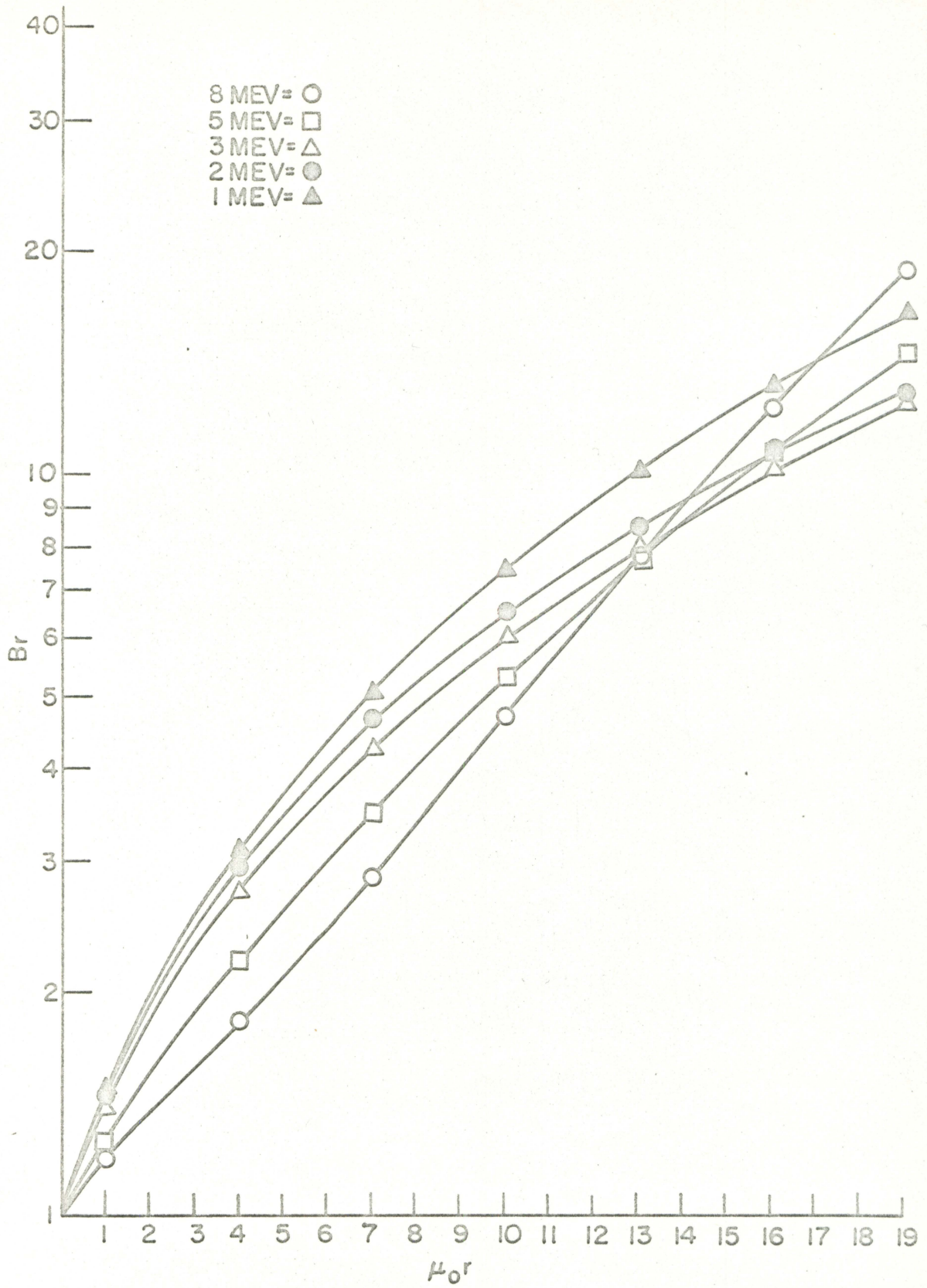


Figure 7. Dose build-up factors for alloy with 45 wt per cent Fe and 55 wt per cent U for gamma ray energies of 1.0, 2.0, 3.0, 5.0 and 8.0 Mev

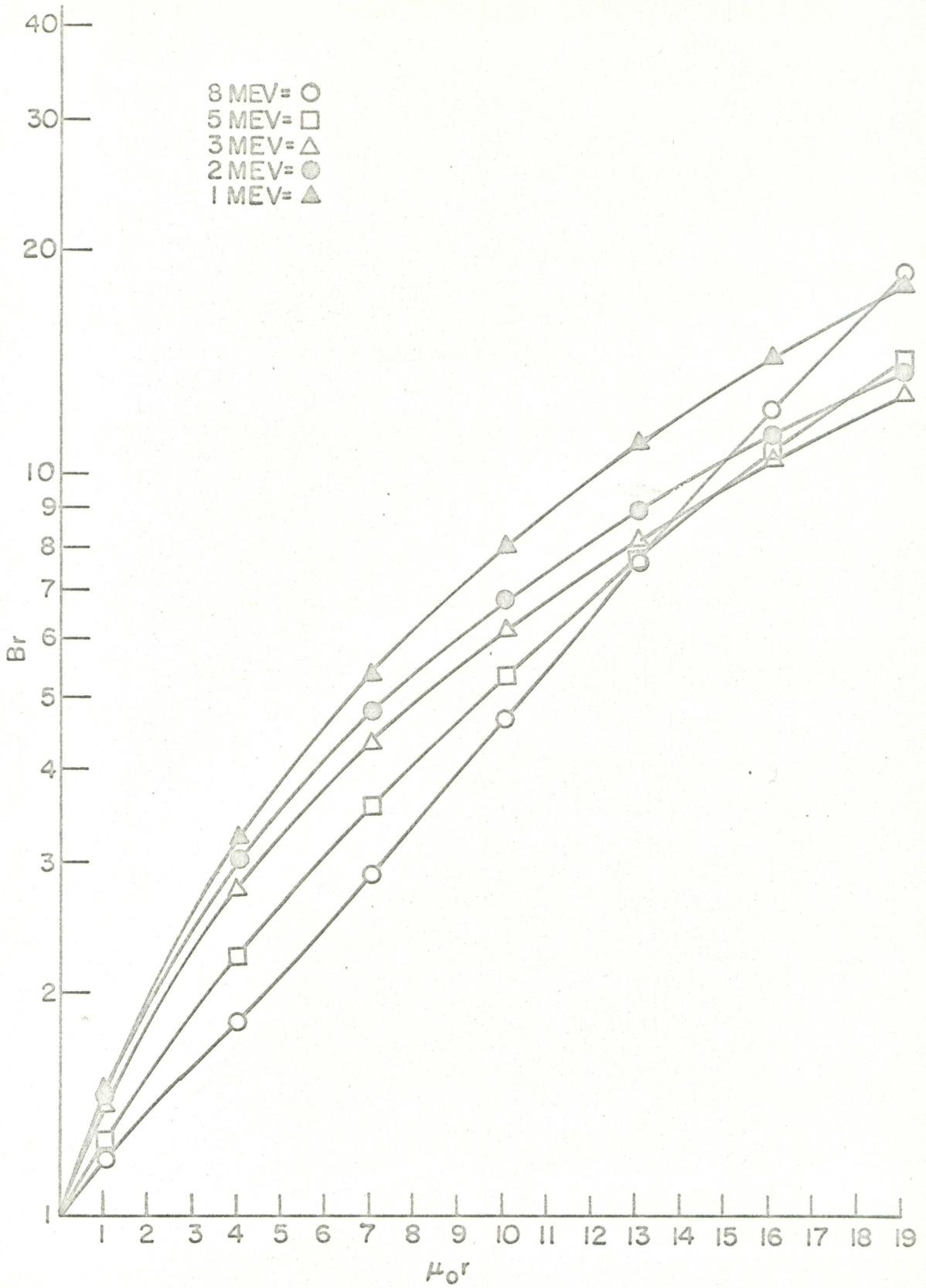


Figure 8. Dose build-up factors for eutectic ( $UFe_2-Fe$ ) for gamma ray energies of 1.0, 2.0, 3.0, 5.0 and 8.0 Mev

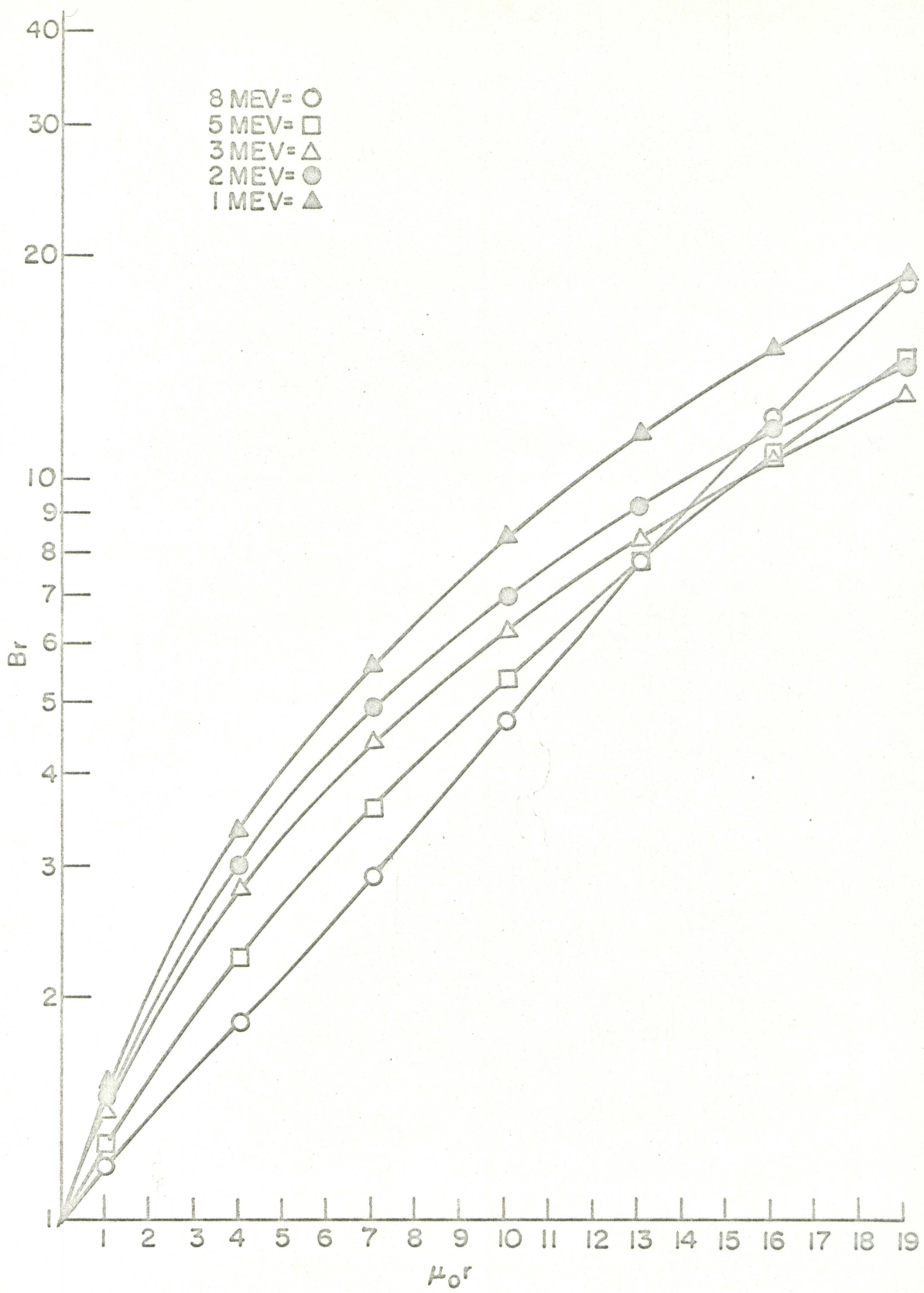




Figure 9. Dose build-up factors for Fe for gamma ray energies of 1.0, 2.0, 3.0, 5.0 and 8.0 Mev

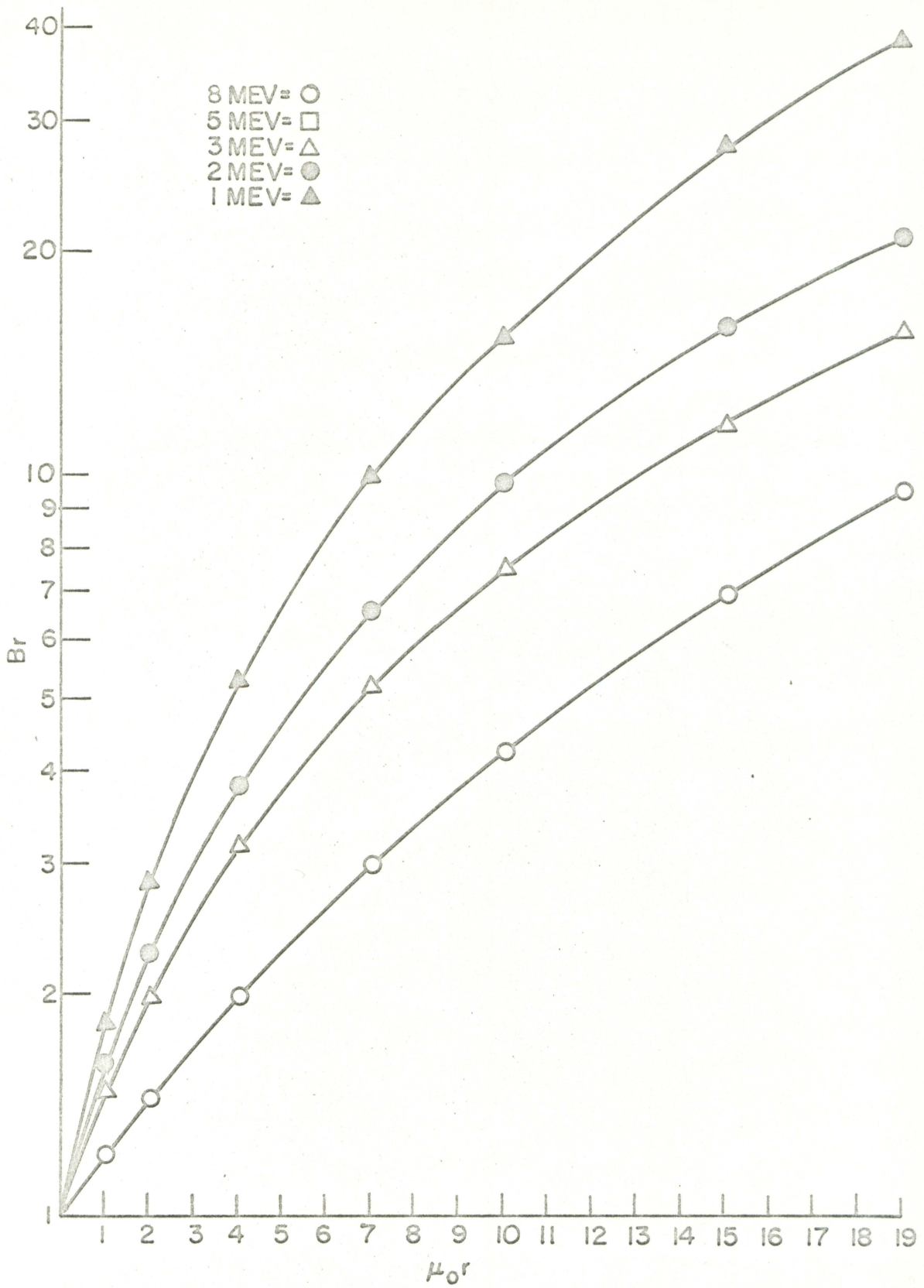


Figure 10. Dose build-up factors for Pb for gamma ray energies of 1.0, 2.0, 3.0, 5.0 and 8.0 Mev

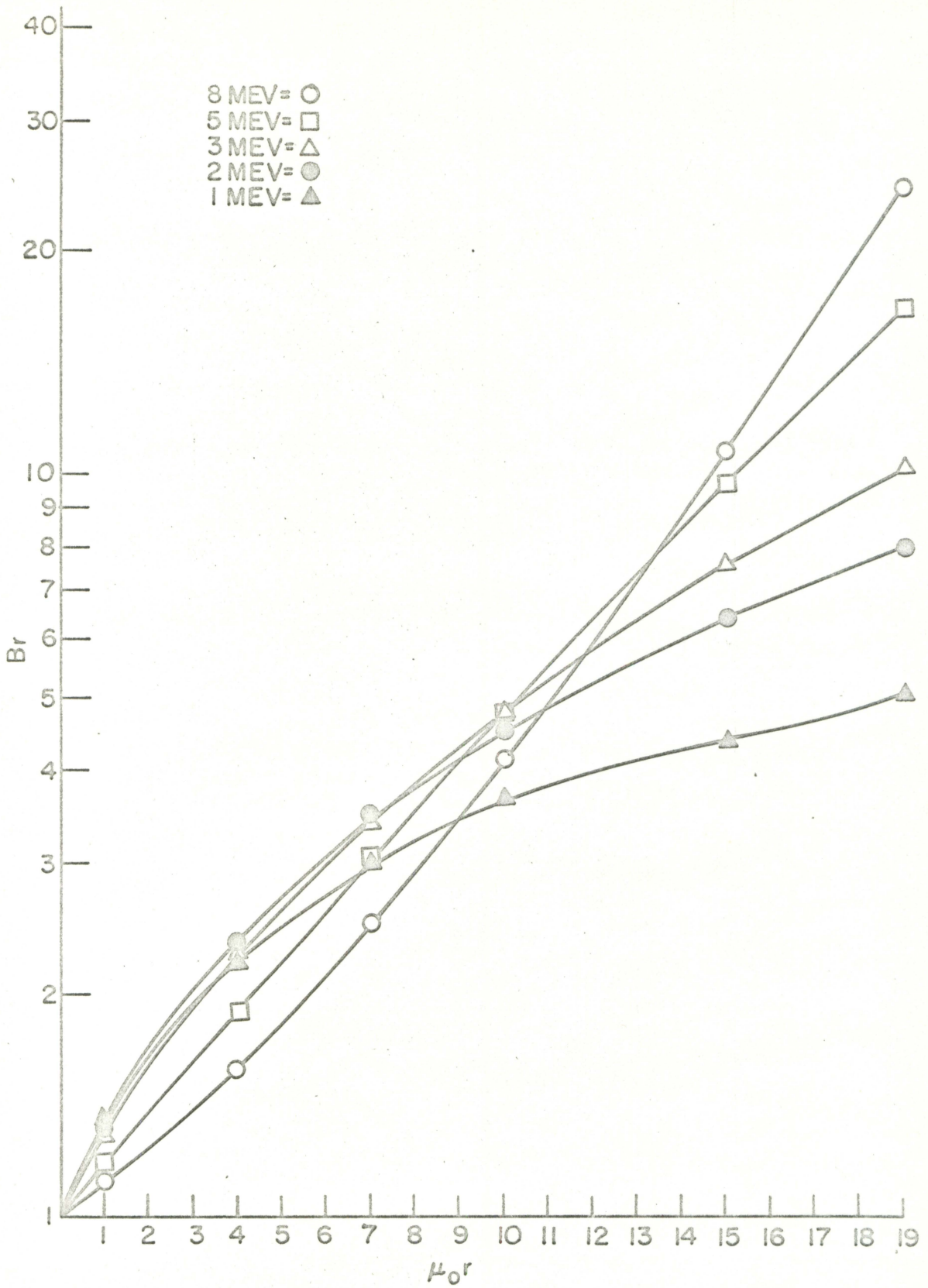
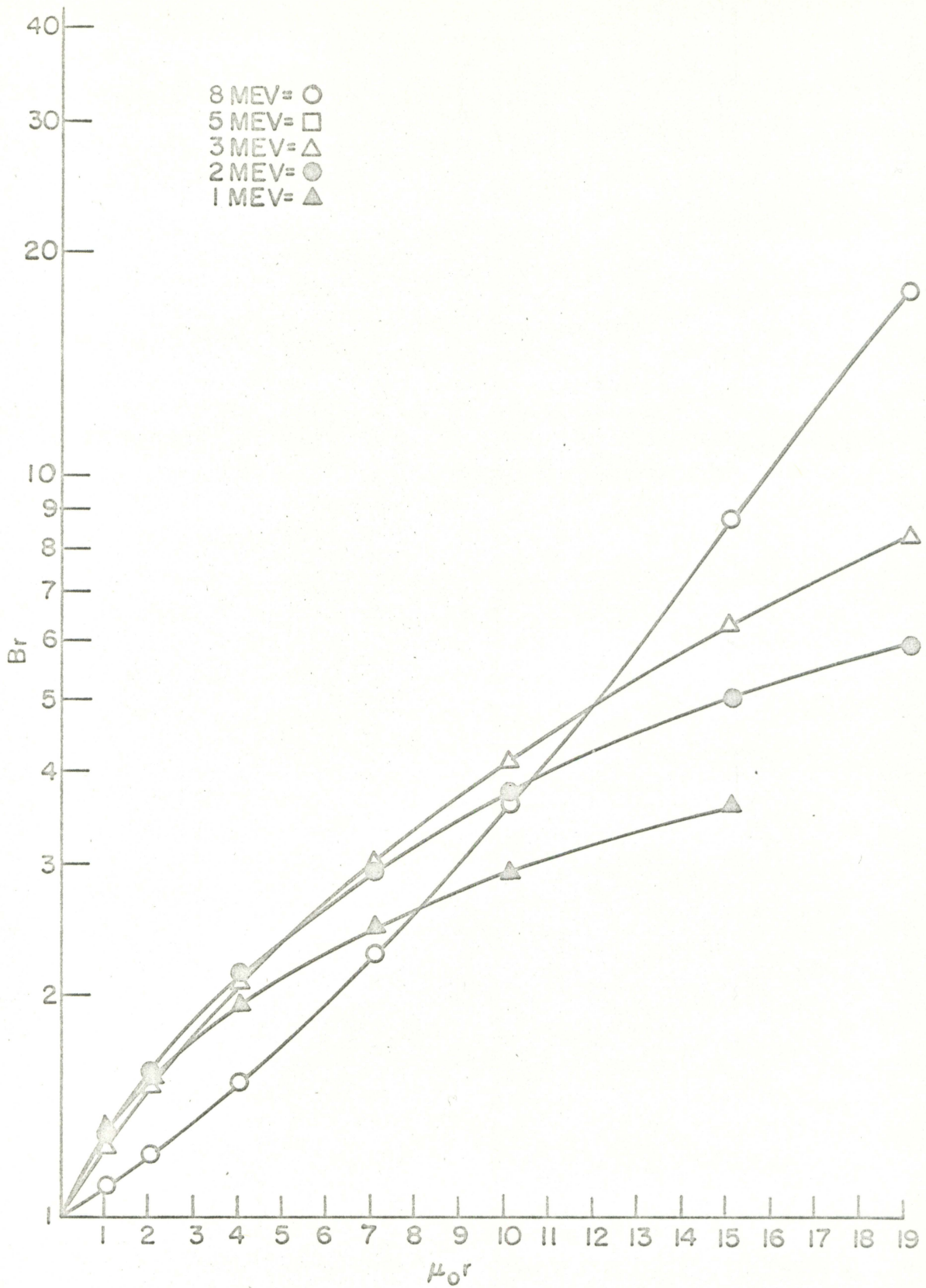


Figure 11. Dose build-up factors for U for gamma ray energies of 1.0, 2.0, 3.0, 5.0 and 8.0 Mev



be greater than that of finite thickness at the same distance. This is due to the fact that in infinite medium there is a probability for back scattering of the photons. In the case of a finite system,  $X$  could be taken at the surface of the shield, and correction factors from infinite to finite medium must be taken into consideration.

The correction factor  $K$  as given by Goldstein (5) is

$$K = \frac{\beta_E(X,X)-1}{\beta_E(X,\infty)-1} \quad (25)$$

where  $\beta(X,\infty)$  and  $\beta(X,X)$  are the build-up factors for an infinite and finite medium, respectively. It was reported in Goldstein (5) that this correction to finite medium is negligible for heavy materials such as lead and uranium, and becomes appreciable only in the light elements at low source energies and short penetrations (less than 4 mean free paths). The correction factor,  $K$ , was plotted graphically. It was shown that it increases by increasing the atomic weight of the shield.

Table 4. Linear attenuation coefficients

Material	Gamma ray energy, Mev				
	1	2	3	5	8
U <sub>6</sub> Fe	1.3303	0.8524	0.7904	0.8011	0.8630
eutectic (U <sub>6</sub> Fe-UFe <sub>2</sub> )	1.1769	0.7590	0.7000	0.7034	0.7530
UFe <sub>2</sub>	0.9317	0.6137	0.5559	0.5437	0.5697
alloy (35 wt per cent Fe + 65 wt per cent U)	0.8460	0.5596	0.5051	0.4913	0.5125
alloy (40 wt per cent Fe + 60 wt per cent U)	0.7982	0.5302	0.4769	0.4612	0.4790
alloy (45 wt per cent Fe + 55 wt per cent U)	0.7578	0.5057	0.4530	0.4354	0.4498
eutectic (UFe <sub>2</sub> -Fe)	0.7372	0.4935	0.4410	0.4221	0.4346
Fe	0.4646	0.3314	0.2807	0.3509	0.2316
Pb	0.7769	0.5164	0.4810	0.4836	0.5357
U	1.4434	0.9216	0.8572	0.8538	0.9429



Table 5. Coefficients for polynomial build-up factor equation

Material	Gamma ray energy, Mev	$\alpha_1$	$\alpha_2 \times 10^{-2}$	$\alpha_3 \times 10^{-4}$
$U_6Fe$	1	0.28373	-0.98472	0.35063
	2	0.33832	-0.34750	0.18829
	3	0.30633	0.40575	0.20589
	5	0.18542	0.92353	0.99789
	8	0.16598	0.12624	0.31790
eutectic ( $U_6Fe-UFe_2$ )	1	0.31411	-0.46560	0.28981
	2	0.35878	-0.90794	0.14769
	3	0.31998	0.53489	0.17214
	5	0.19496	0.97098	0.94143
	8	0.16984	-0.14489	0.30458
$UFe_2$	1	0.40957	0.12448	0.74886
	2	0.42339	0.70000	0.23671
	3	0.36302	0.95982	0.60094
	5	0.22574	0.11237	0.75628
	8	0.18164	-0.74513	0.25875
alloy (35 wt per cent Fe + 65 wt per cent U)	1	0.42840	0.15938	0.29229
	2	0.43606	0.85307	-0.16411
	3	0.37158	0.10454	0.36969
	5	0.23347	0.11529	0.71886
	8	0.18460	-0.66769	0.24937
alloy (40 wt per cent Fe + 60 wt per cent U)	1	0.44816	0.19333	-0.12084
	2	0.44909	0.10025	-0.22194
	3	0.38049	0.11269	0.16012
	5	0.24003	0.11862	0.69729
	8	0.18741	-0.58386	0.23955
alloy (45 wt per cent Fe + 55 wt per cent U)	1	0.46949	0.23324	-0.64398
	2	0.46330	0.11739	-0.48366
	3	0.39046	0.12201	-0.82487
	5	0.19054	-0.26211	0.66489
	8	0.19054	-0.48538	0.22814
eutectic ( $UFe_2-Fe$ )	1	0.48328	0.25733	-0.93460
	2	0.47211	0.12792	-0.64595
	3	0.39620	0.12856	-0.26332
	5	0.25255	0.12443	0.60746
	8	0.19261	-0.42593	0.22108
Fe	1	0.78696	0.81149	-0.81256

Table 5 (Continued).

Material	Gamma ray energy, Mev	$\alpha_1$	$\alpha_2$ $\times 10^{-2}$	$\alpha_3$ $\times 10^{-4}$
Fe	2	0.66363	0.35684	-0.42033
	3	0.52671	0.26053	-0.36078
	5	0.43220	0.19413	-0.17918
	8	0.25161	0.13566	0.19467
Pb	1	0.36810	-0.11858	0.29907
	2	0.38051	-0.42238	0.71372
	3	0.32012	0.95374	0.13813
	5	0.25326	0.12435	0.65656
	8	0.24034	-0.42838	0.61569
U	1	0.31383	-0.18486	0.63817
	2	0.31904	-0.23750	0.10616
	3	0.27583	-0.65278	0.97184
	5	0.22773	0.97844	0.40626
	8	0.17819	0.21152	0.38031

Table 6. Mass absorption coefficients

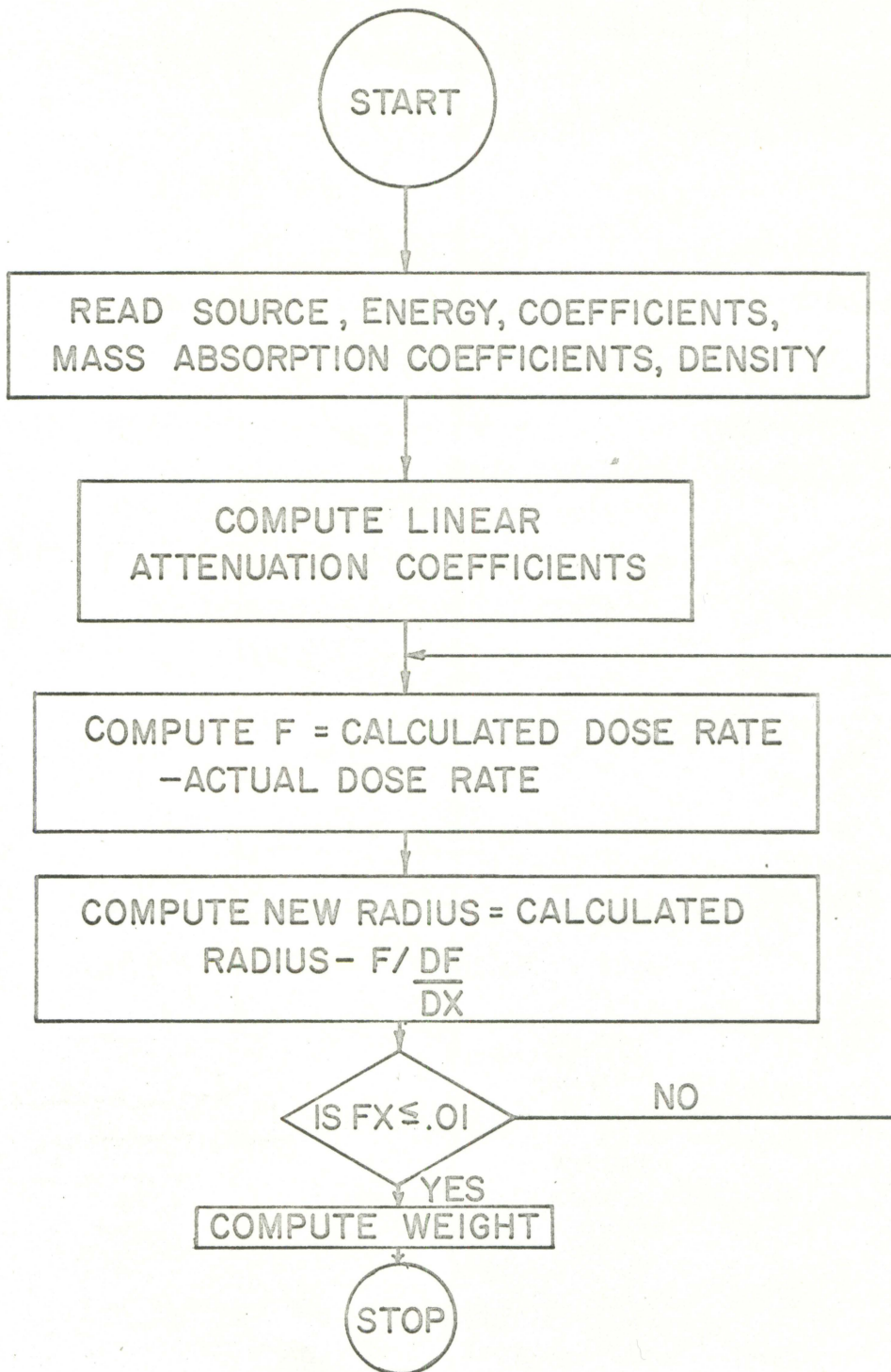
Material	Gamma ray energy, Mev				
	1	2	3	5	8
U <sub>6</sub> Fe	.0751	.0481	.04466	.04526	.04876
eutectic (U <sub>6</sub> Fe-UFe <sub>2</sub> )	.07401	.04774	.04403	.04424	.04736
UFe <sub>2</sub>	.07058	.04625	.04212	.04116	.04316
alloy (35 wt per cent Fe + 65 wt per cent U)	.06991	.04625	.04175	.04061	.04236
alloy (40 wt per cent Fe + 60 wt per cent U)	.06928	.04603	.04140	.04004	.04158
alloy (45 wt per cent Fe + 55 wt per cent U)	.06857	.04577	.04100	.03941	.04071
eutectic (UFe <sub>2</sub> -Fe)	.06813	.04561	.04076	.03902	.04017
Fe	.05957	.04250	.03600	.04500	.02970
Pb	.06844	.04550	.04238	.04261	.04720
U	.07576	.04838	.04500	.04482	.04950

The dose rate equation

$$D = 5.24 \times 10^6 \frac{EC}{R^2} \beta(\mu r) e^{-\mu r} \text{ mr/hr}$$

was used to calculate the required radius R that gives a dose rate between 9.9 and 10.1 mr/hr on the shield surface. The equation was solved by using the IBM 360/50 computer. R was taken equal to r in the above equation, since the dose rate wanted is at the surface of the sphere. The gamma ray energies used were 1, 2, 3, 5 and 8 Mev. The source strengths of 1, 10, 100, 100 and 5000 curies were used for the five different energies. The values of the linear absorption coefficients  $\mu$  were taken from Table 4 and the values of the constants  $\alpha_1$ ,  $\alpha_2$  and  $\alpha_3$  were taken from Table 5. For the computer calculations R was assigned an initial value of 5 cm, and then varied by using the Newton-Ralphson interaction method until the dose rate fell between 9.1 and 10.1 mr/hr. After the determination of the required value of R, the weight of the shield W was calculated. The above procedure was repeated to find the weight and thickness of Fe, Pb and U which would give the same dose. Figure 12 shows a flow diagram of the program that was used for the calculations.

Figure 12. Computer flow diagram



## RESULTS AND DISCUSSION

The results obtained from the computer program are displayed as curves of the shield radius and weight versus the source strength in curies. Comparison between the shield weight for some of the alloys and the elements iron, lead and uranium, also are shown on curves. The shield radius versus the source strength curves (Figures 13 through 19) show nearly straight line variation on the semi log plot over the whole region of investigation. Therefore, one is able to determine accurately the shield thickness required for the intermediate source strengths. The increase of radius resulting from decreasing the amount of U in the system can be noticed for the same energy and the same source strength.

From the shield weight curves (Figures 20 through 26) comparison can be made between the alloys and iron, lead and uranium. From these curves it can be seen that gamma radiation with initial energy 5.0 Mev requires the maximum amount of shielding only in case the shield is  $U_6Fe$  or an eutectic ( $U_6Fe-UFe_2$ ). For all the other compounds under investigation, it can be seen that gamma rays with initial energy 8.0 Mev require the maximum amount of shielding.

In Figures 27, 28 and 29 one can compare the shield weights of  $Pb$  lead, uranium,  $U_6Fe$ , eutectic ( $U_6Fe-UFe_2$ ),  $UFe_2$ , alloy with 40 wt per cent Fe + 60 wt per cent U and the eutectic ( $UFe_2-Fe$ ). The curves are a smoothly varying function and are similar in shape. From Figures 27 and 28 one can find that the shield weights needed using  $U_6Fe$  eutectic ( $U_6Fe-UFe_2$ ) or  $UFe_2$  fall within the region between U and  $Pb$  the shield weights needed by the other compositions fall within the region between  $Pb$  and Fe.

Figure 13.  $U_6$ Fe shield radius requirements for gamma ray energies 1.0, 2.0, 3.0, 5.0 and 8.0 Mev



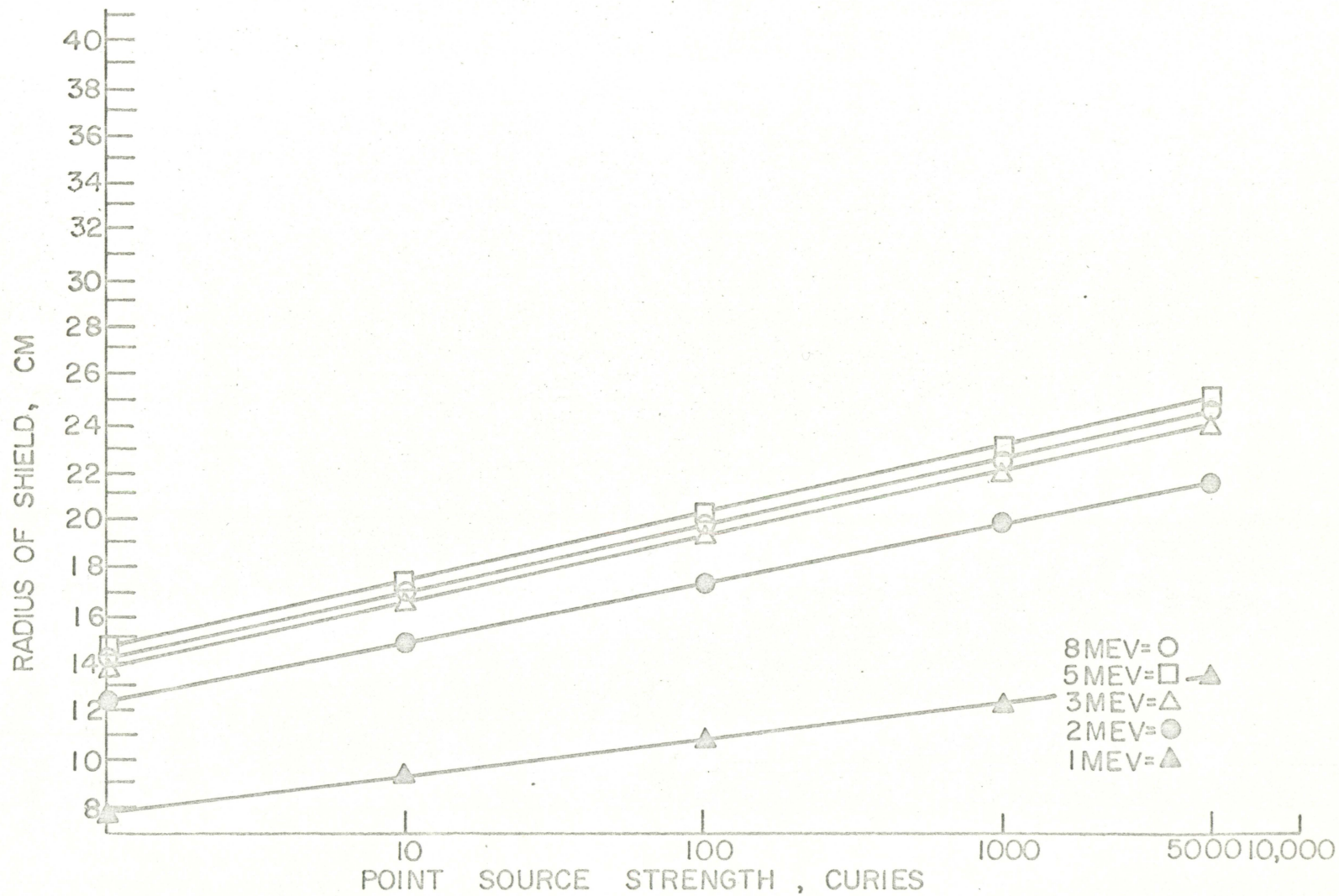


Figure 14. Eutectic ( $U_6Fe-UFe_2$ ) shield radius requirements for gamma ray energies 1.0, 2.0, 3.0, 5.0 and 8.0 Mev

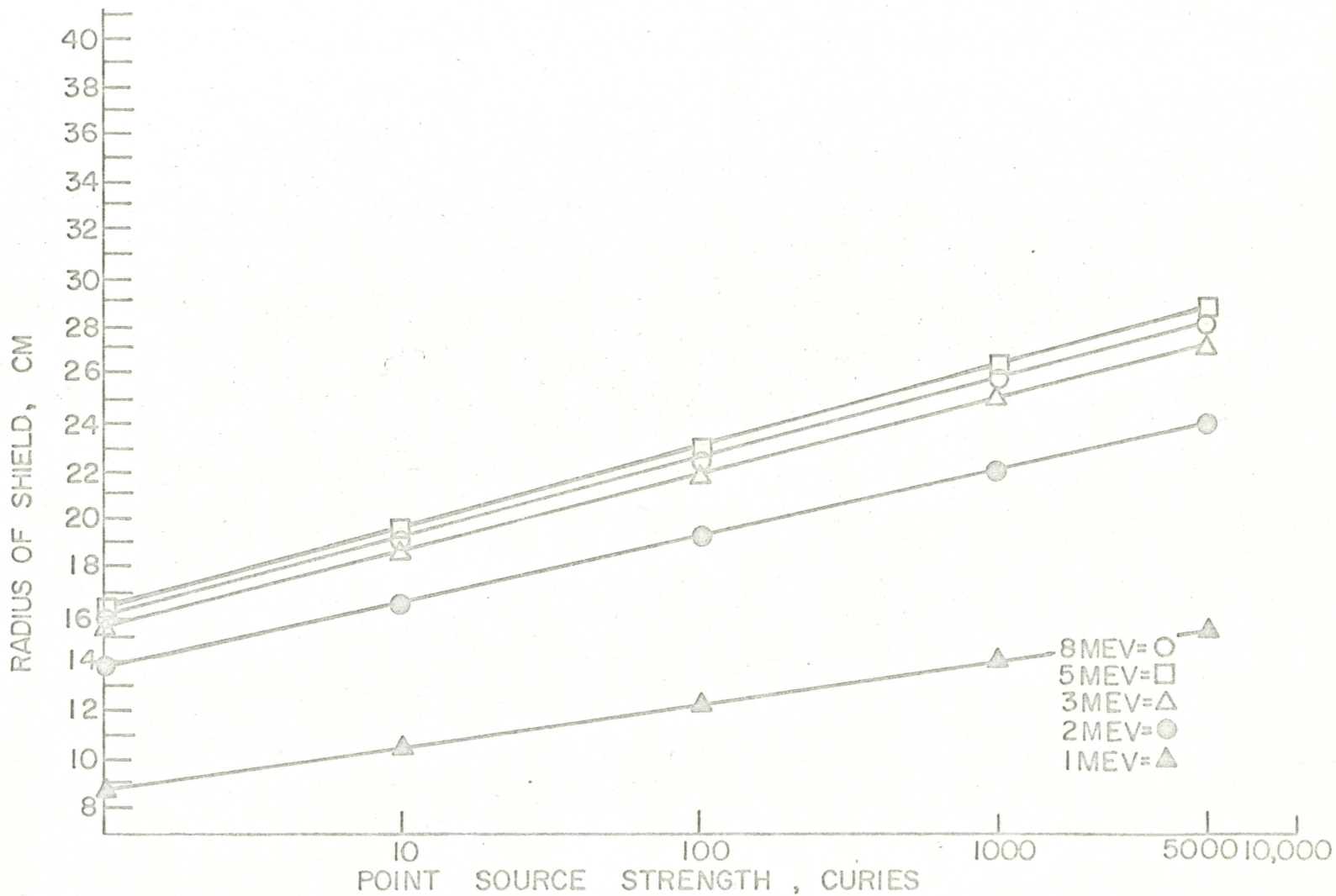


Figure 15.  $UFe_2$  shield radius requirements for gamma ray energies 1.0, 2.0, 3.0, 5.0 and 8.0 Mev

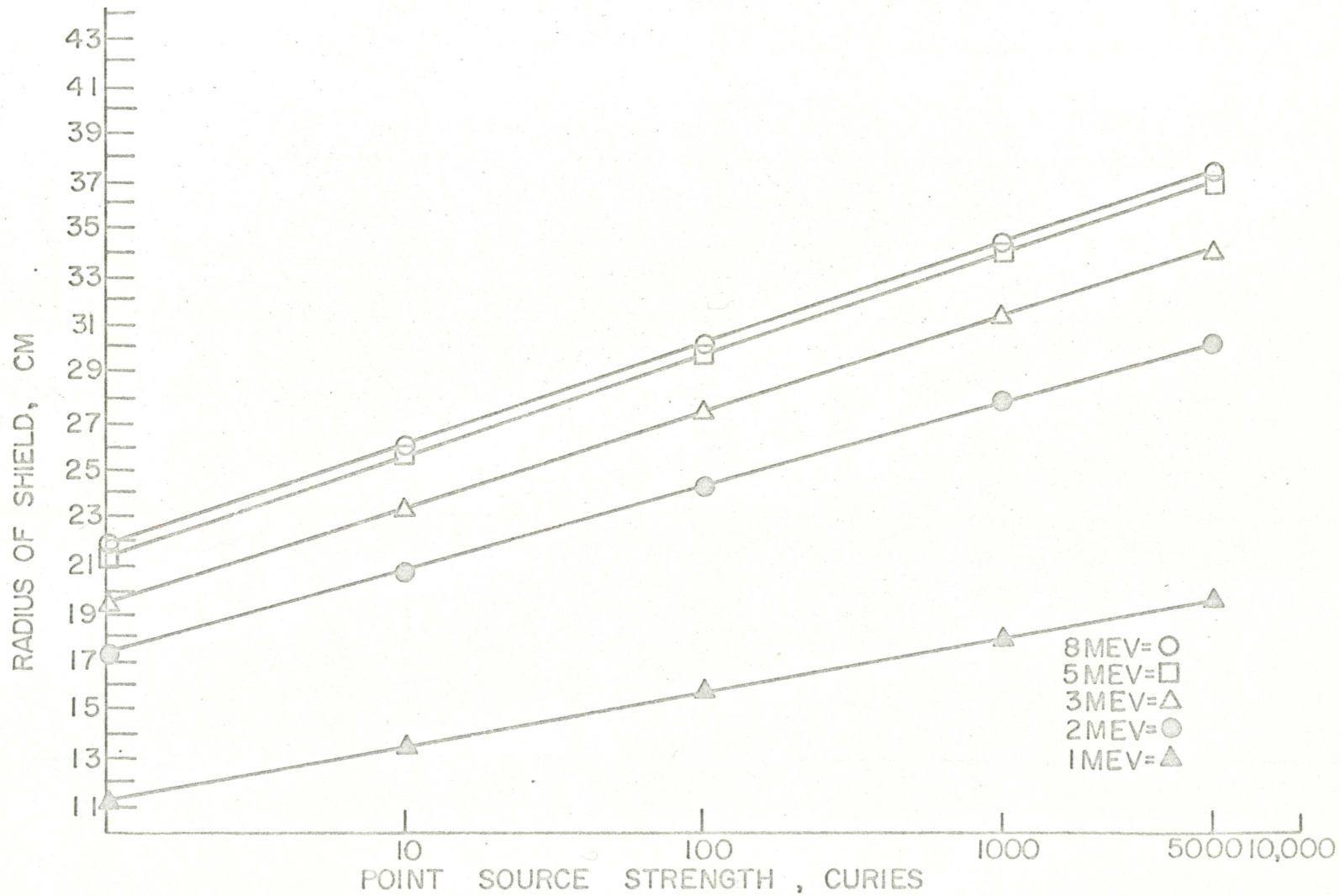


Figure 16. Alloy with 35 wt per cent Fe and 65 wt per cent U shield radius requirements for gamma ray energies 1.0, 2.0, 3.0, 5.0 and 8.0 Mev

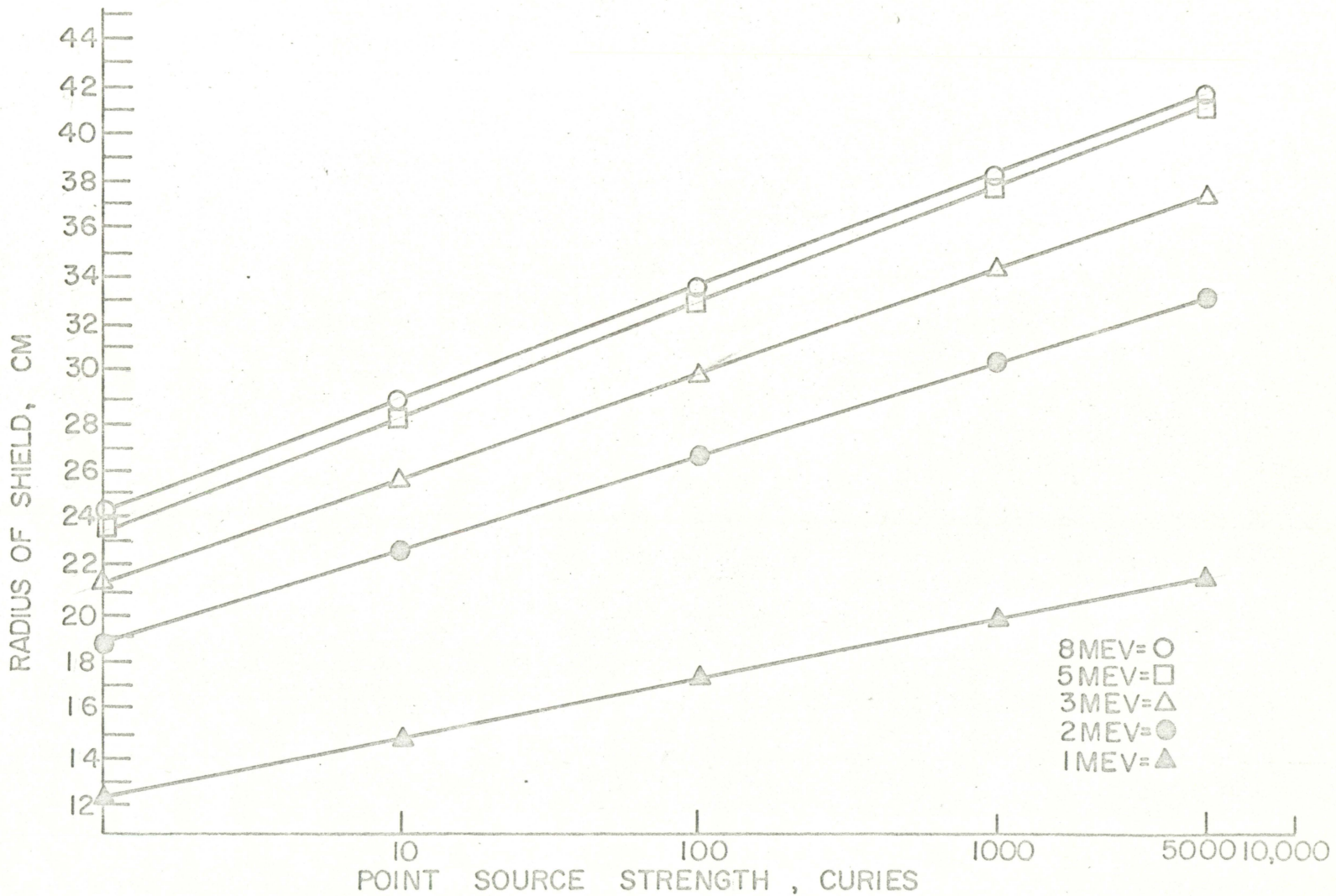


Figure 17. Alloy with 40 wt per cent Fe and 60 wt per cent U shield radius requirements for gamma ray energies 1.0, 2.0, 3.0, 5.0 and 8.0 Mev



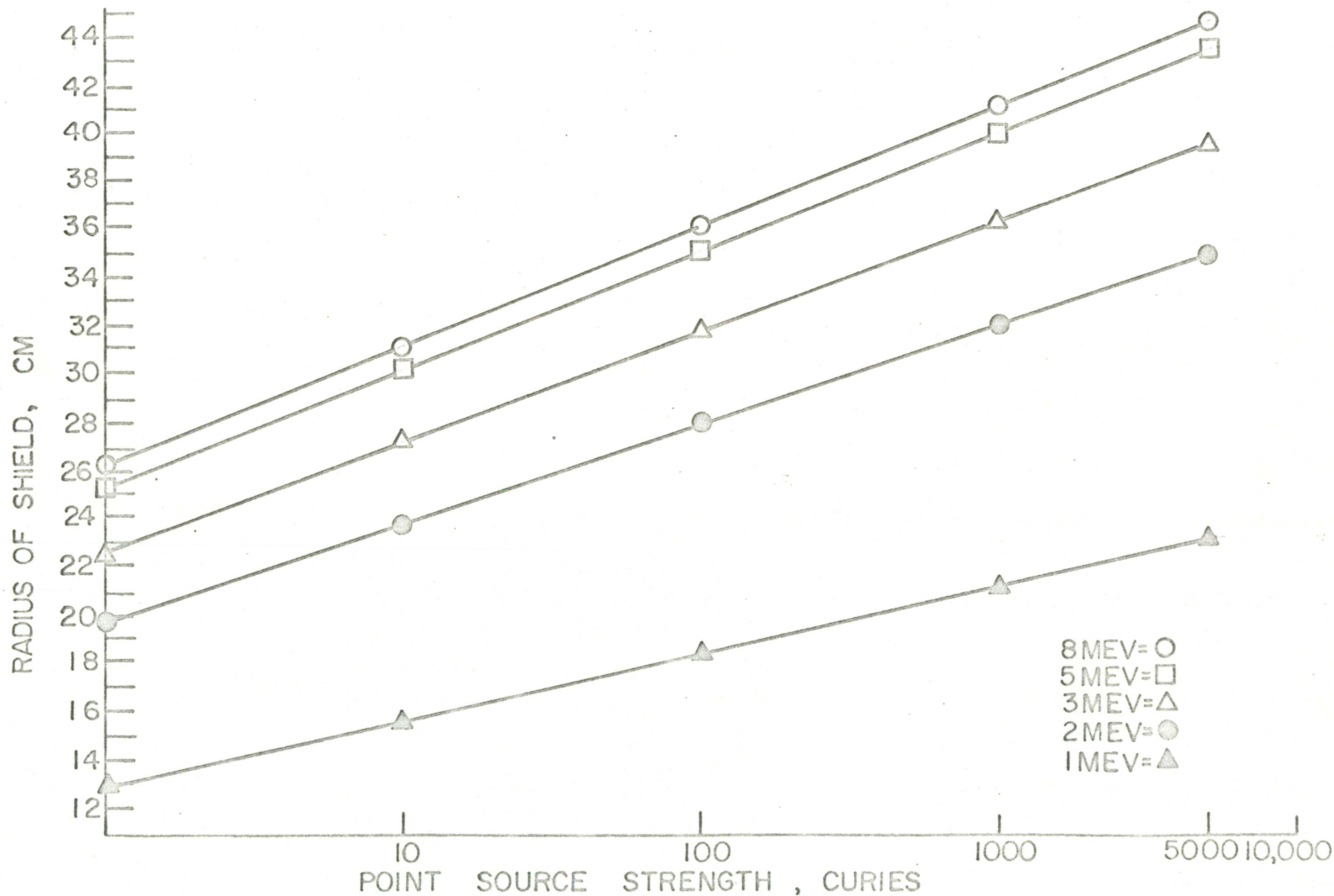


Figure 18. Alloy with 45 wt per cent Fe and 55 wt per cent U shield radius requirements for gamma ray energies 1.0, 2.0, 3.0, 5.0 and 8.0 Mev

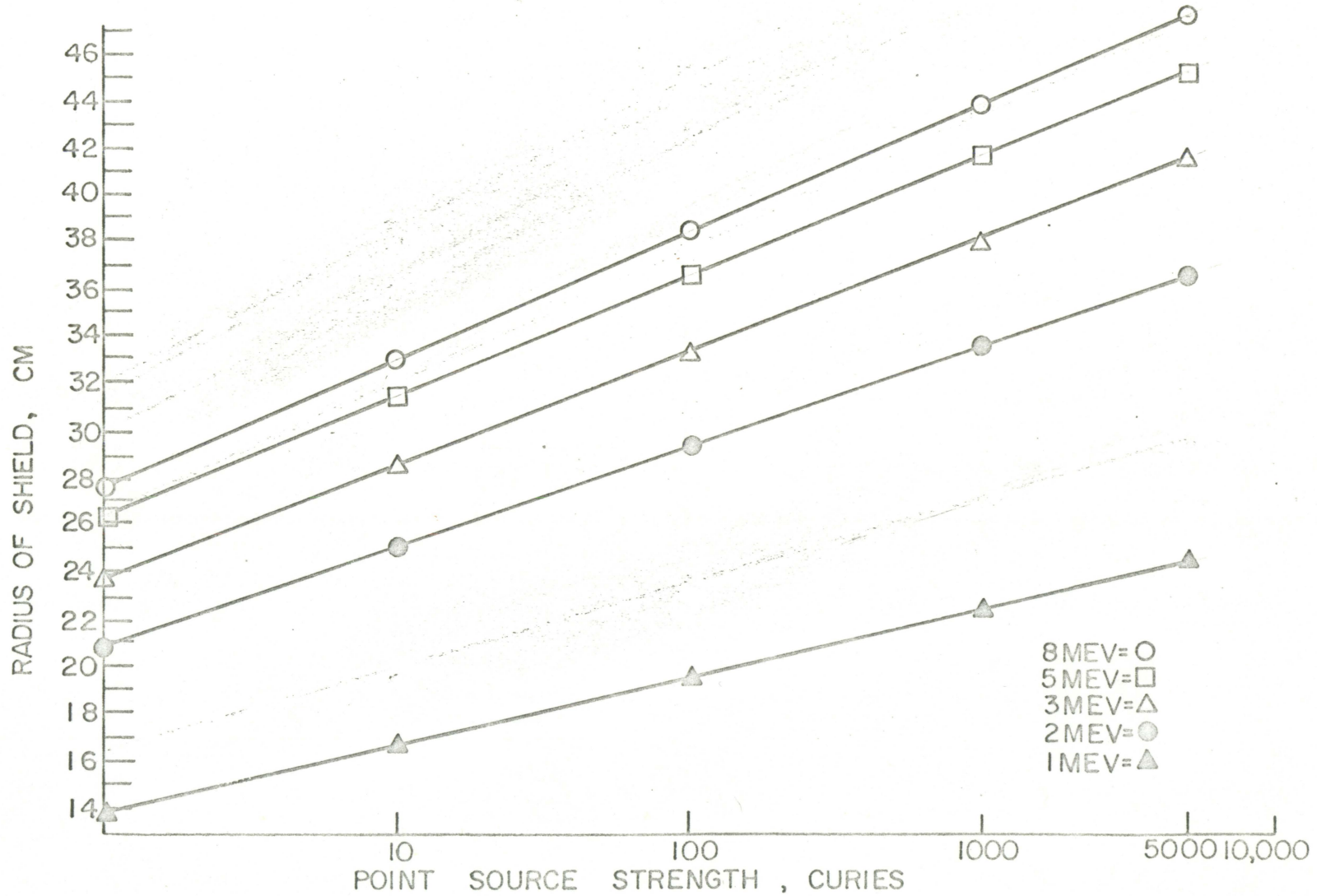


Figure 19. Eutectic ( $UFe_2$ -Fe) shield radius requirements for gamma ray energies 1.0, 2.0, 3.0, 5.0 and 8.0 Mev

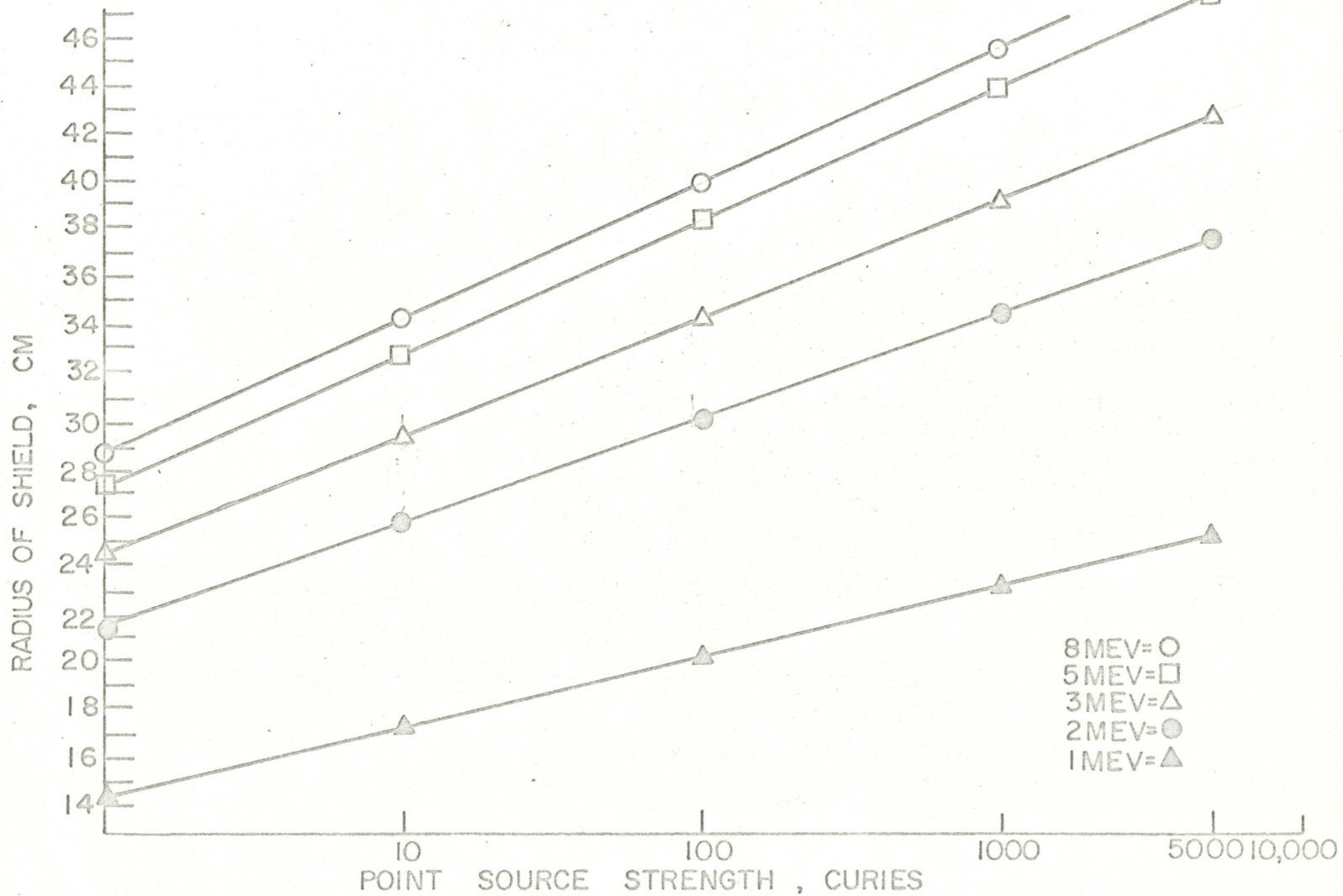


Figure 20.  $U_6Fe$  shield weight requirements for gamma ray energies 1.0, 2.0, 3.0, 5.0 and 8.0 Mev

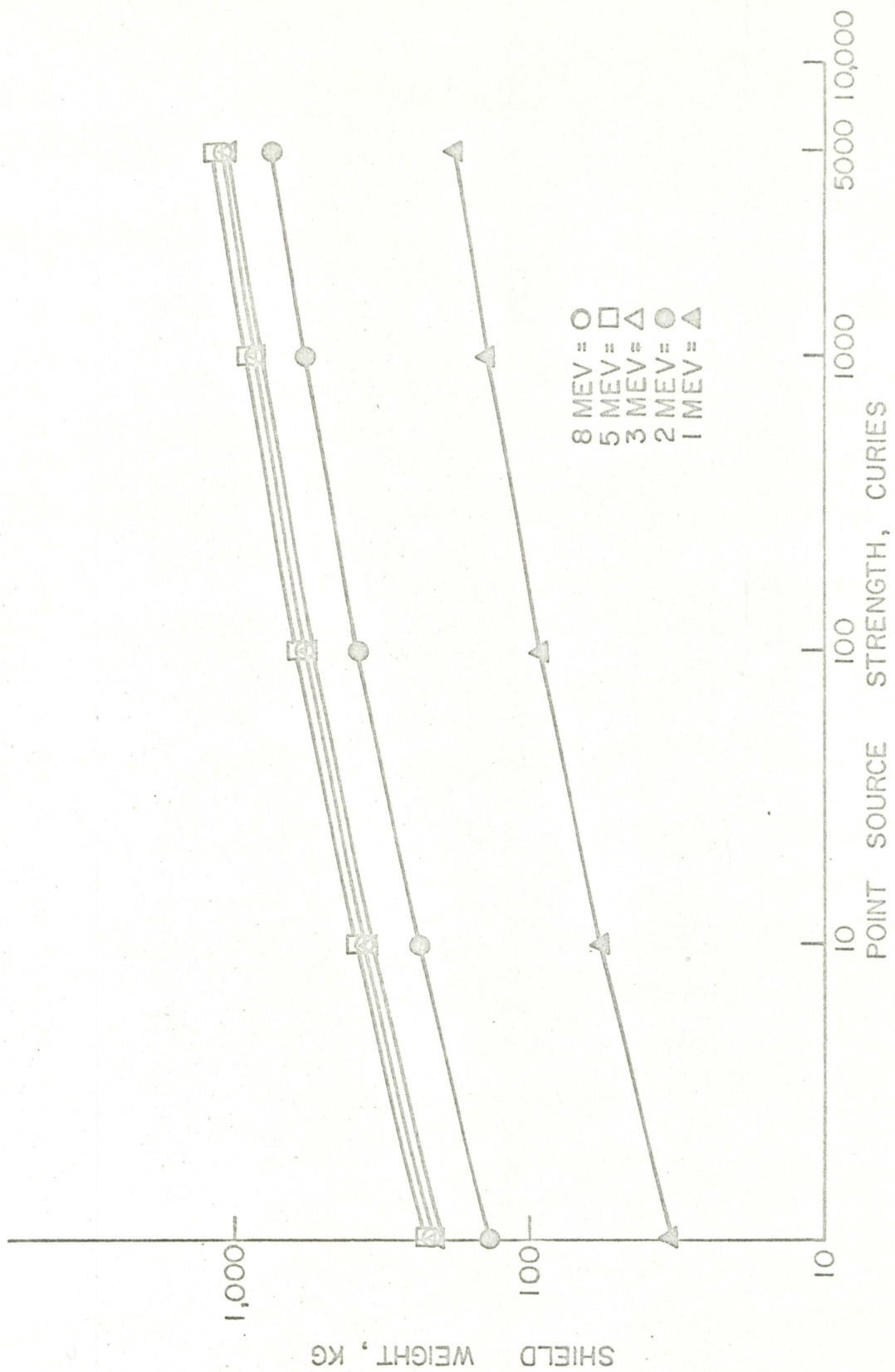


Figure 21. Eutectic ( $U_6Fe-UFe_2$ ) shield weight requirements for gamma ray energies 1.0, 2.0, 3.0, 5.0 and 8.0 Mev



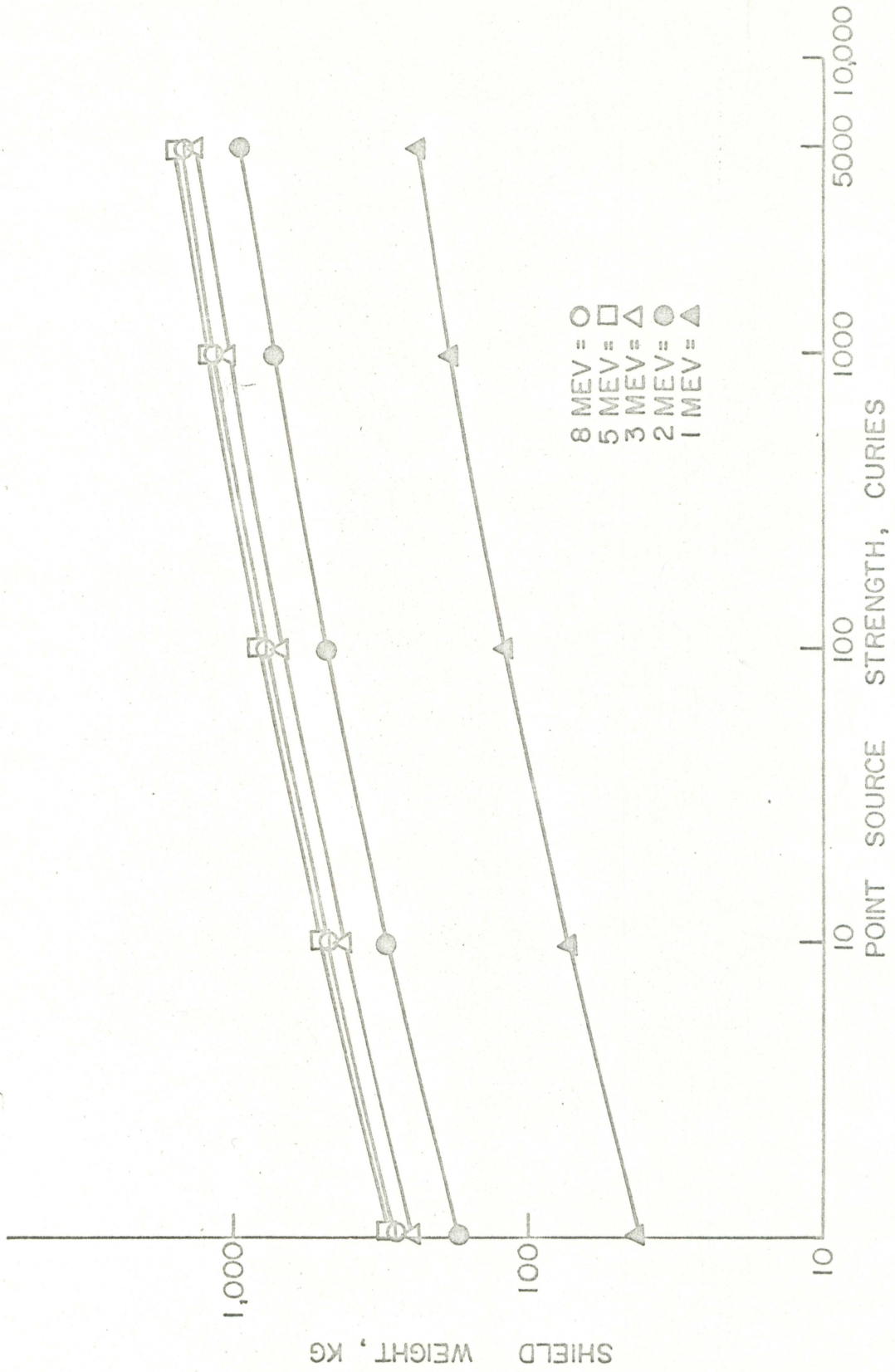


Figure 22.  $UFe_2$  shield weight requirements for gamma ray energies 1.0, 2.0, 3.0, 5.0 and 8.0 Mev.

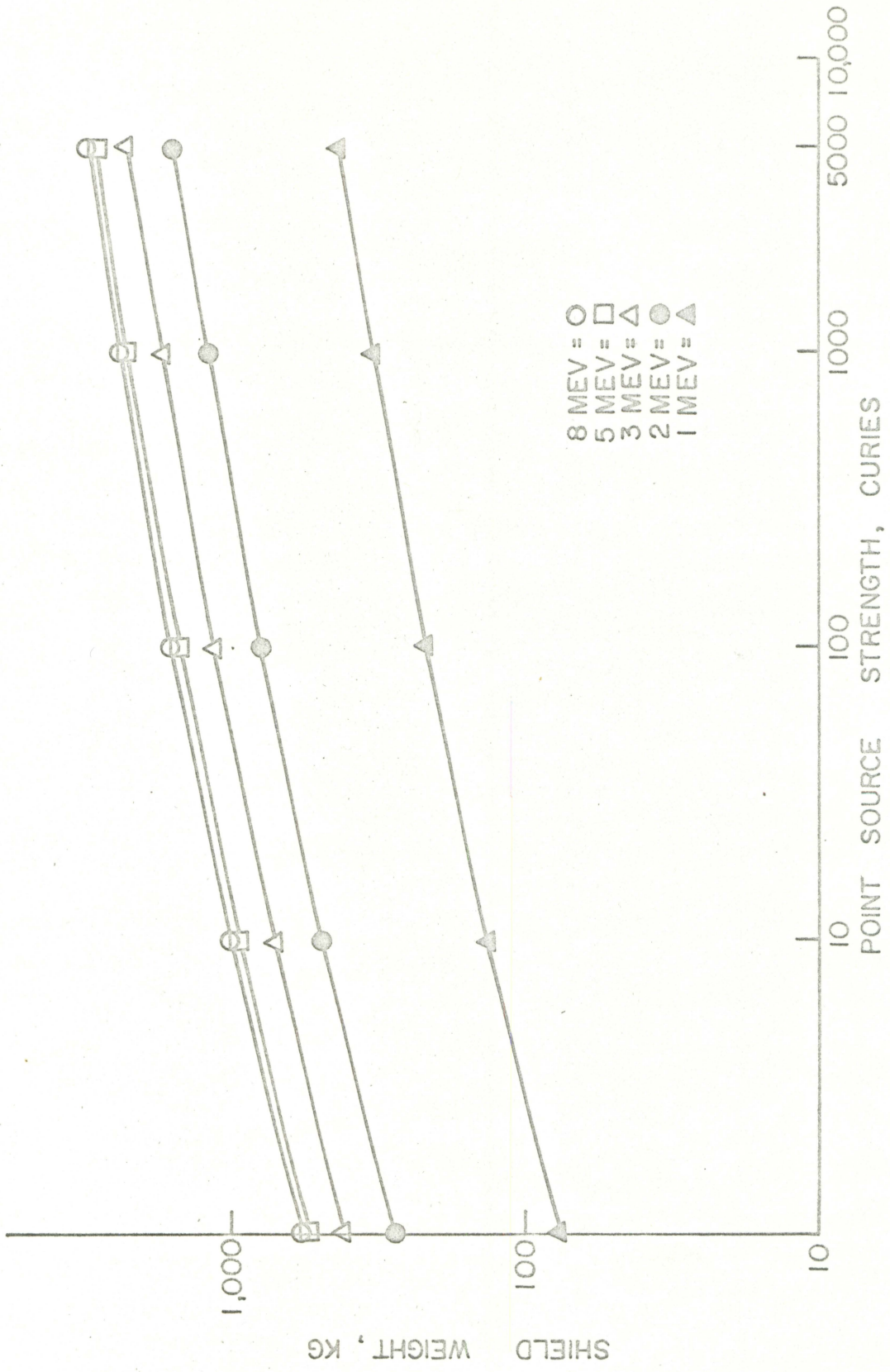


Figure 23. Alloy with 35 wt per cent Fe and 65 wt per cent U shield weight requirements for gamma ray energies 1.0, 2.0, 3.0, 5.0 and 8.0 Mev

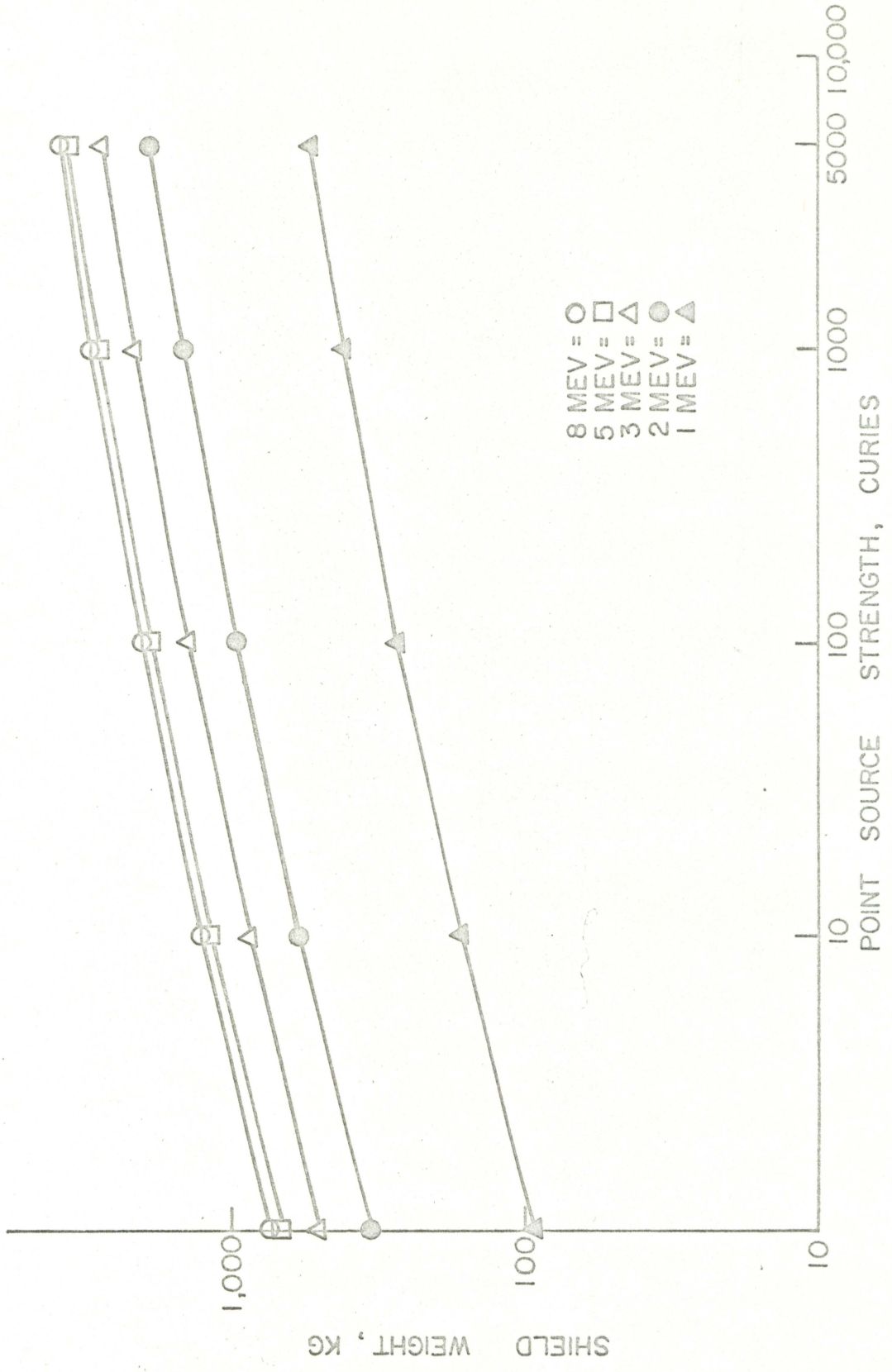


Figure 24. Alloy with 40 wt per cent Fe and 60 wt per cent U shield weight requirements for gamma ray energies 1.0, 2.0, 3.0, 5.0 and 8.0 Mev

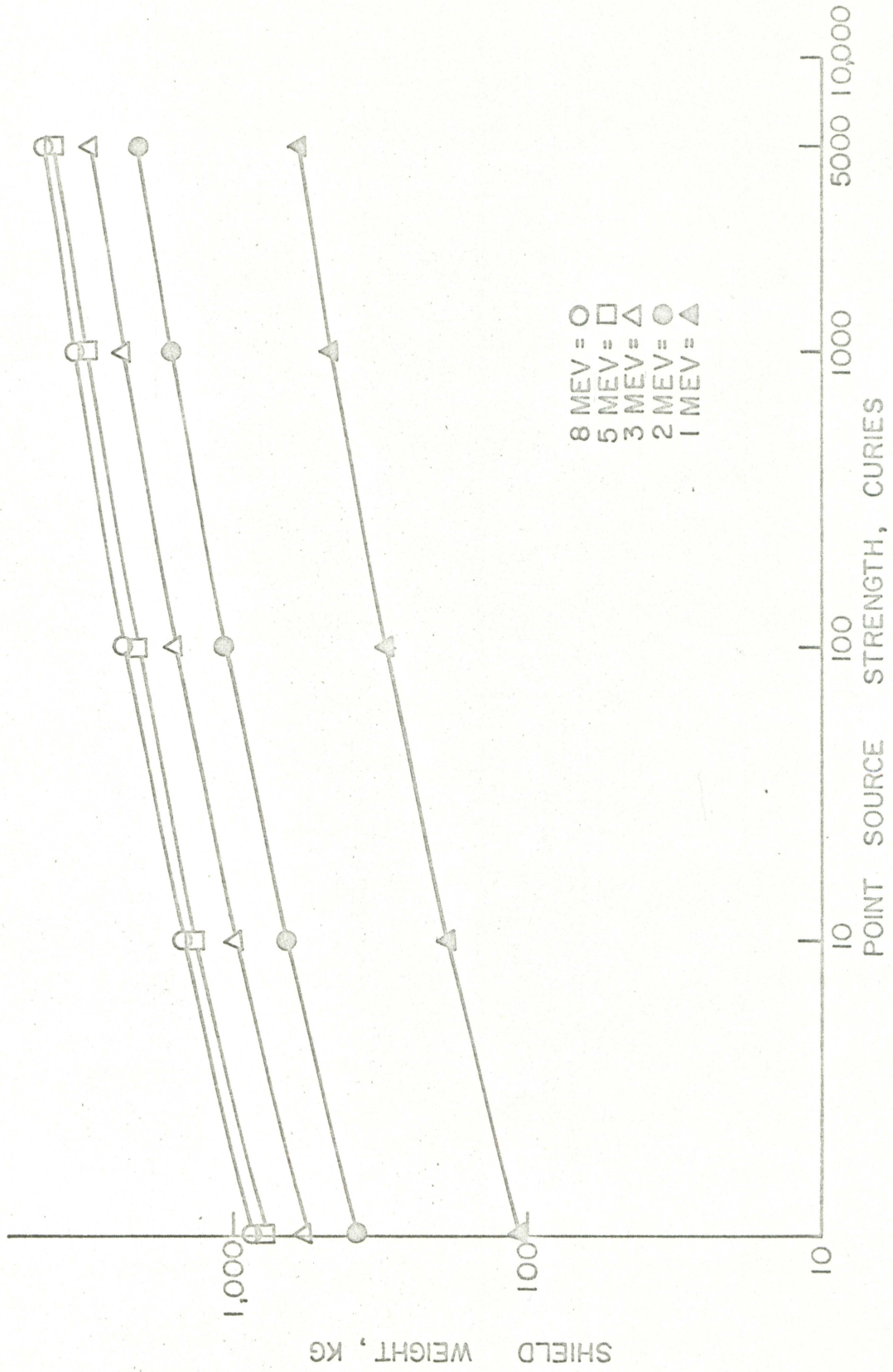


Figure 25. Alloy with 45 wt per cent Fe and 55 wt per cent U shield weight requirements for gamma ray energies 1.0, 2.0, 3.0, 5.0 and 8.0 Mev



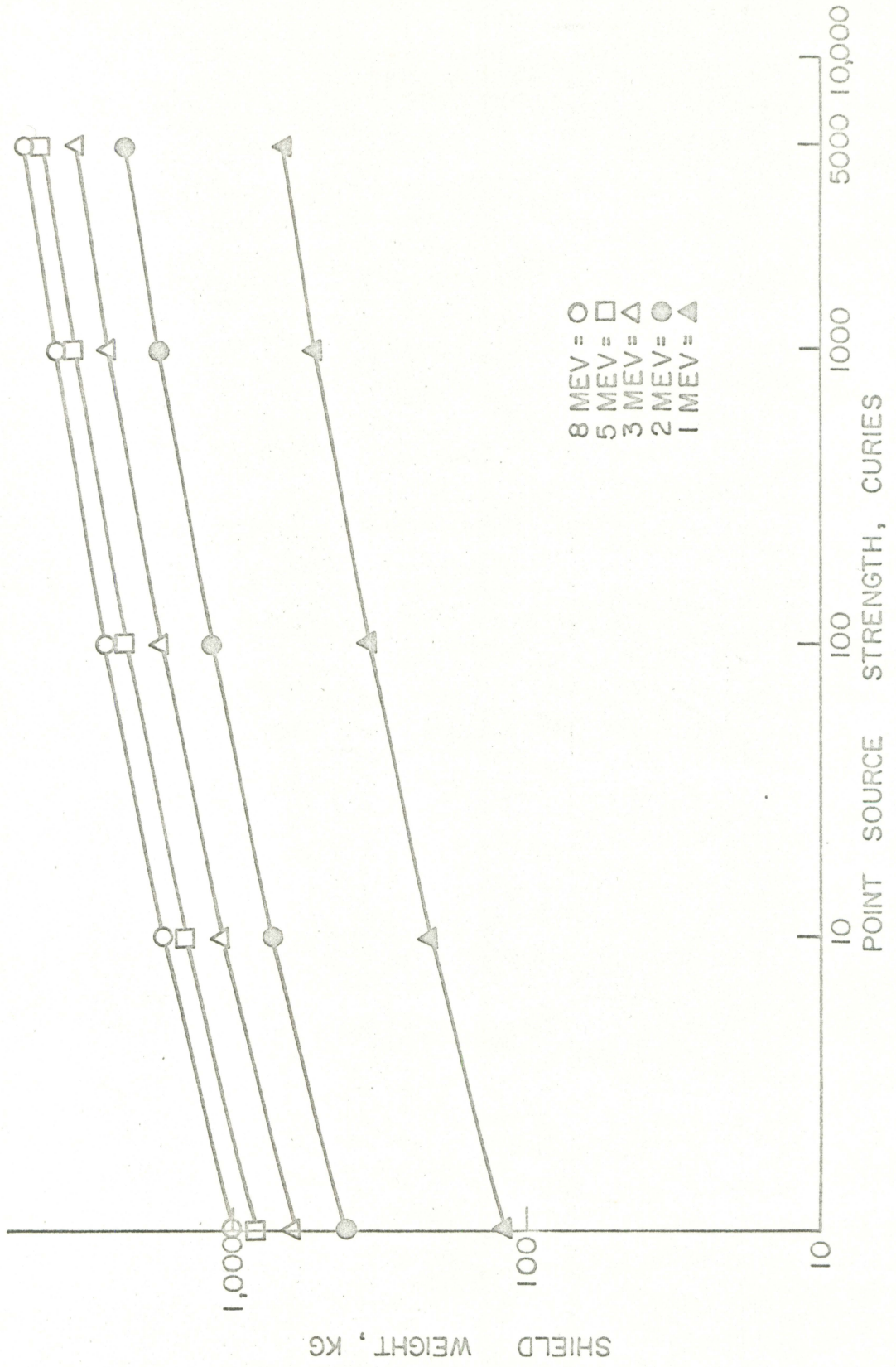


Figure 26. Eutectic ( $\text{UFe}_2$ -Fe) shield weight requirements for gamma ray energies 1.0, 2.0, 3.0, 5.0 and 8.0 Mev

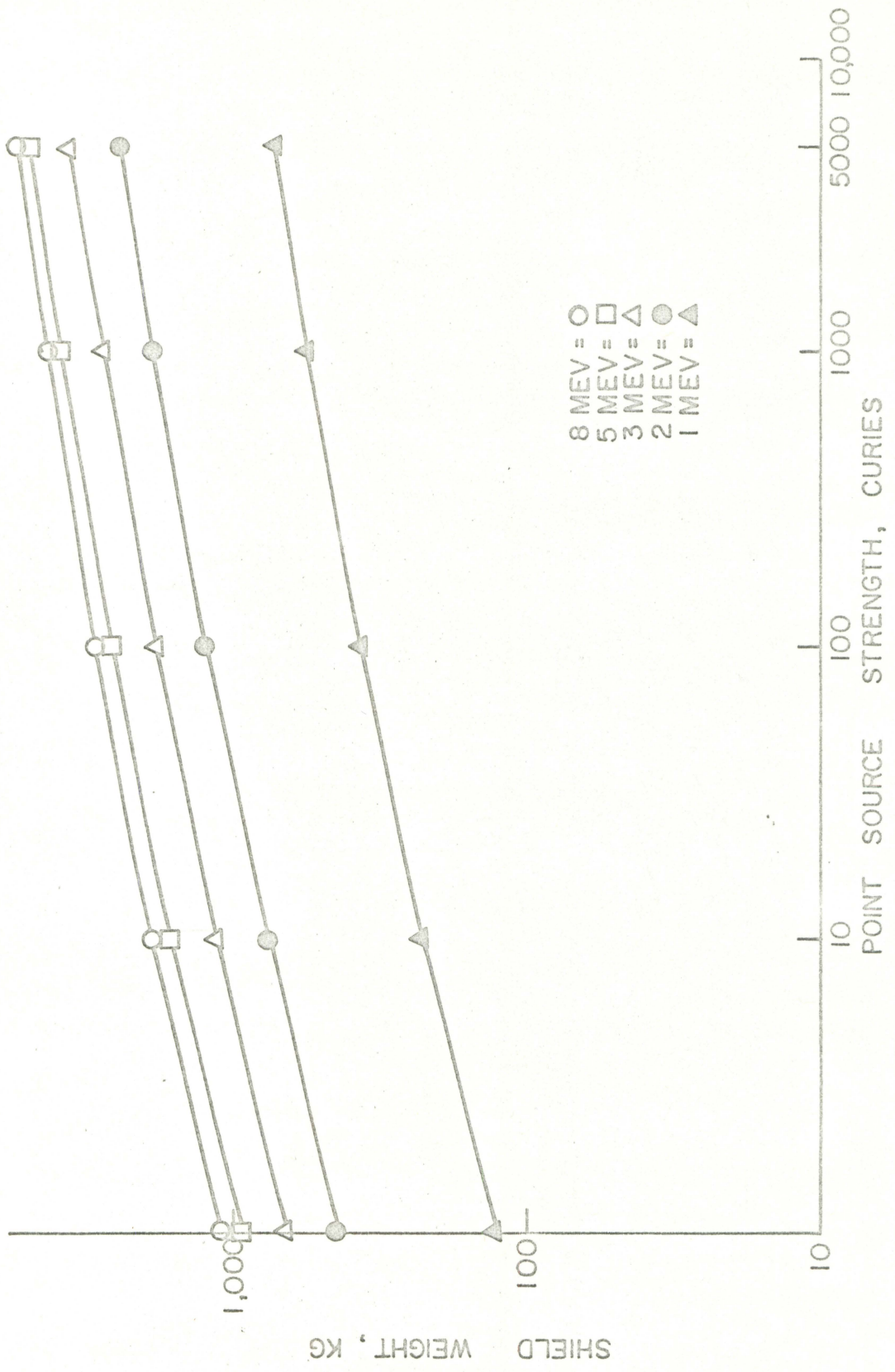


Figure 27. Shield weight requirements of selected materials for gamma rays of energy 1 Mev

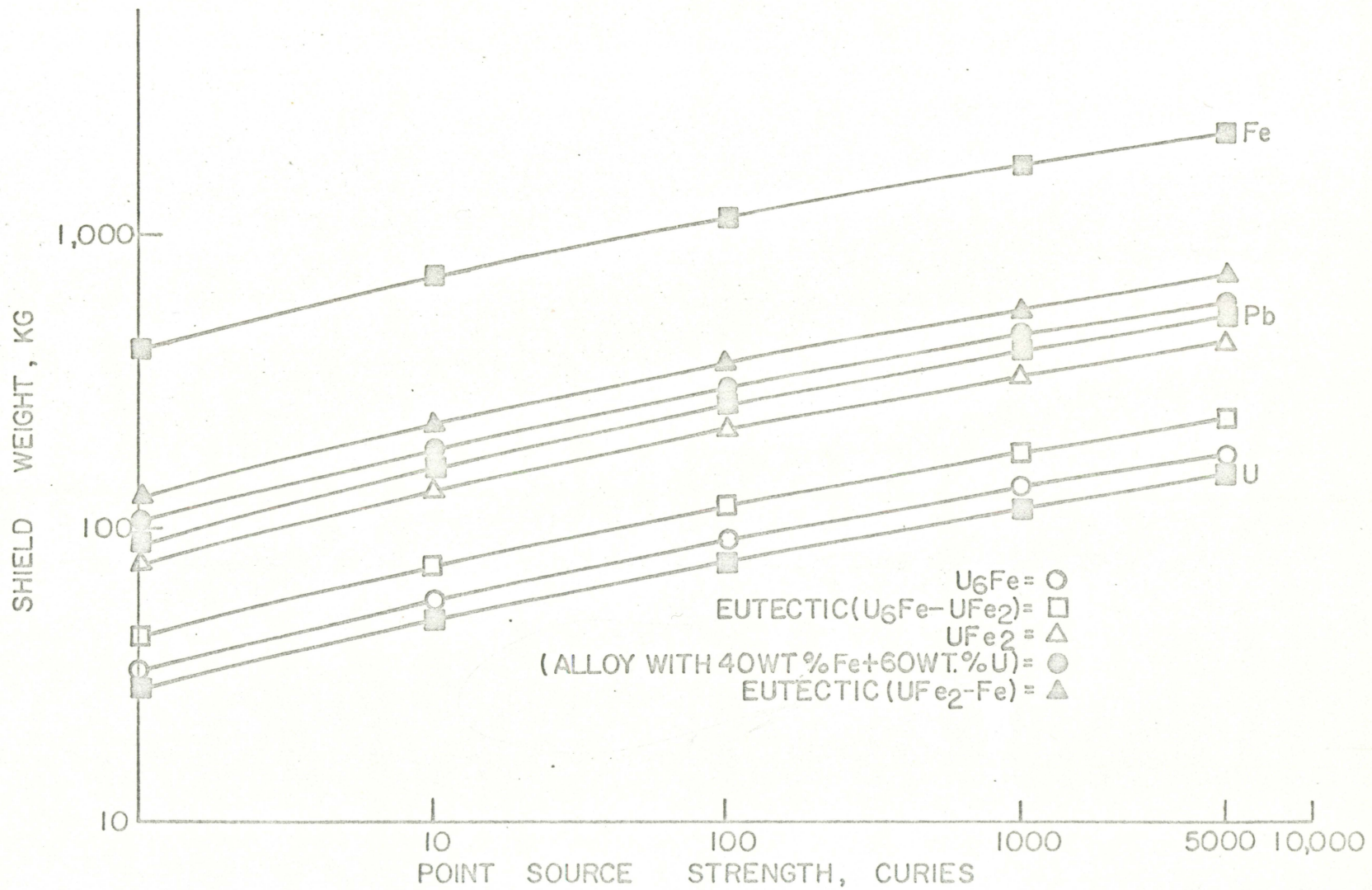


Figure 28. Shield weight requirements of selected material for gamma rays of energy 2 Mev

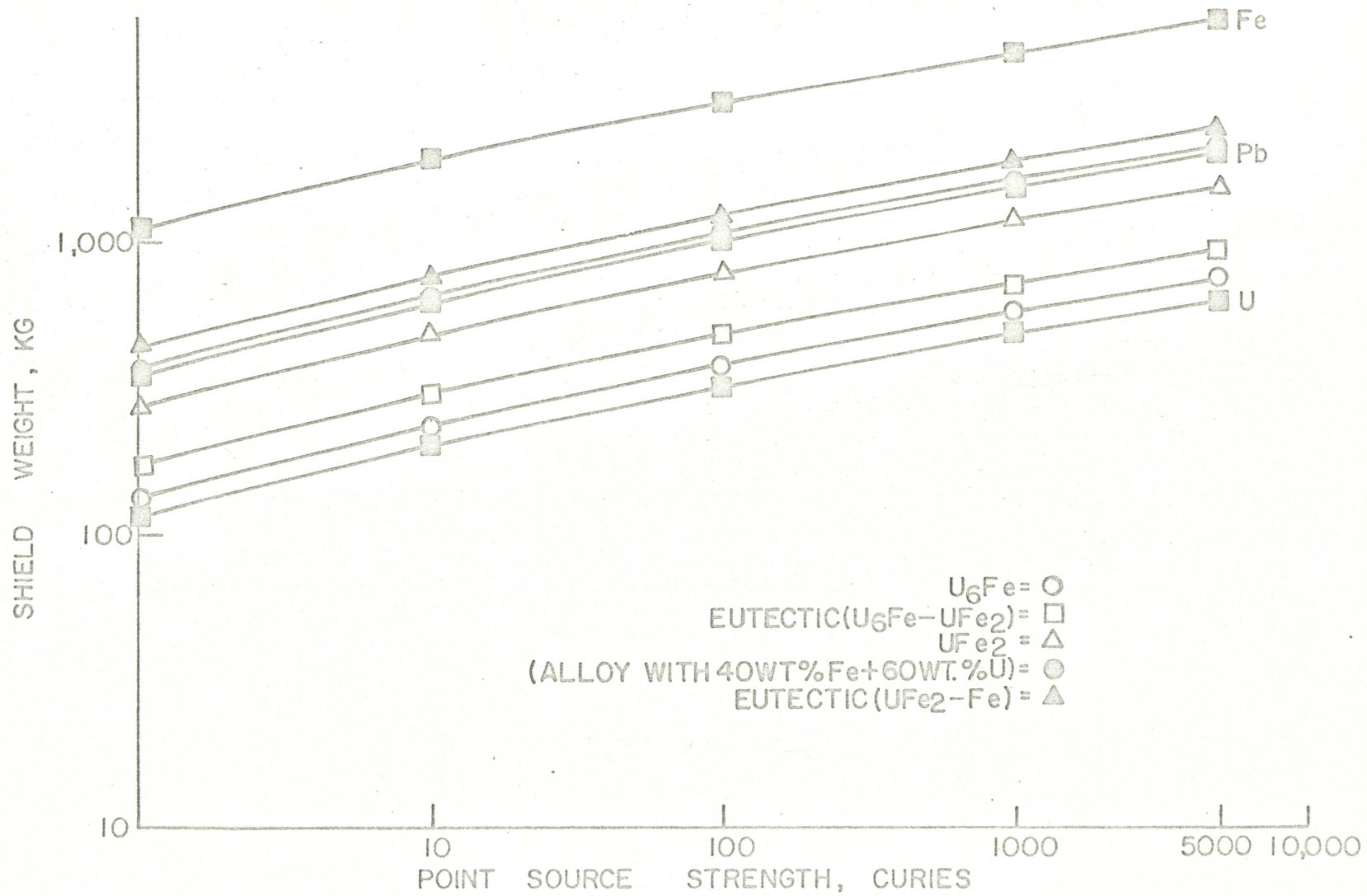
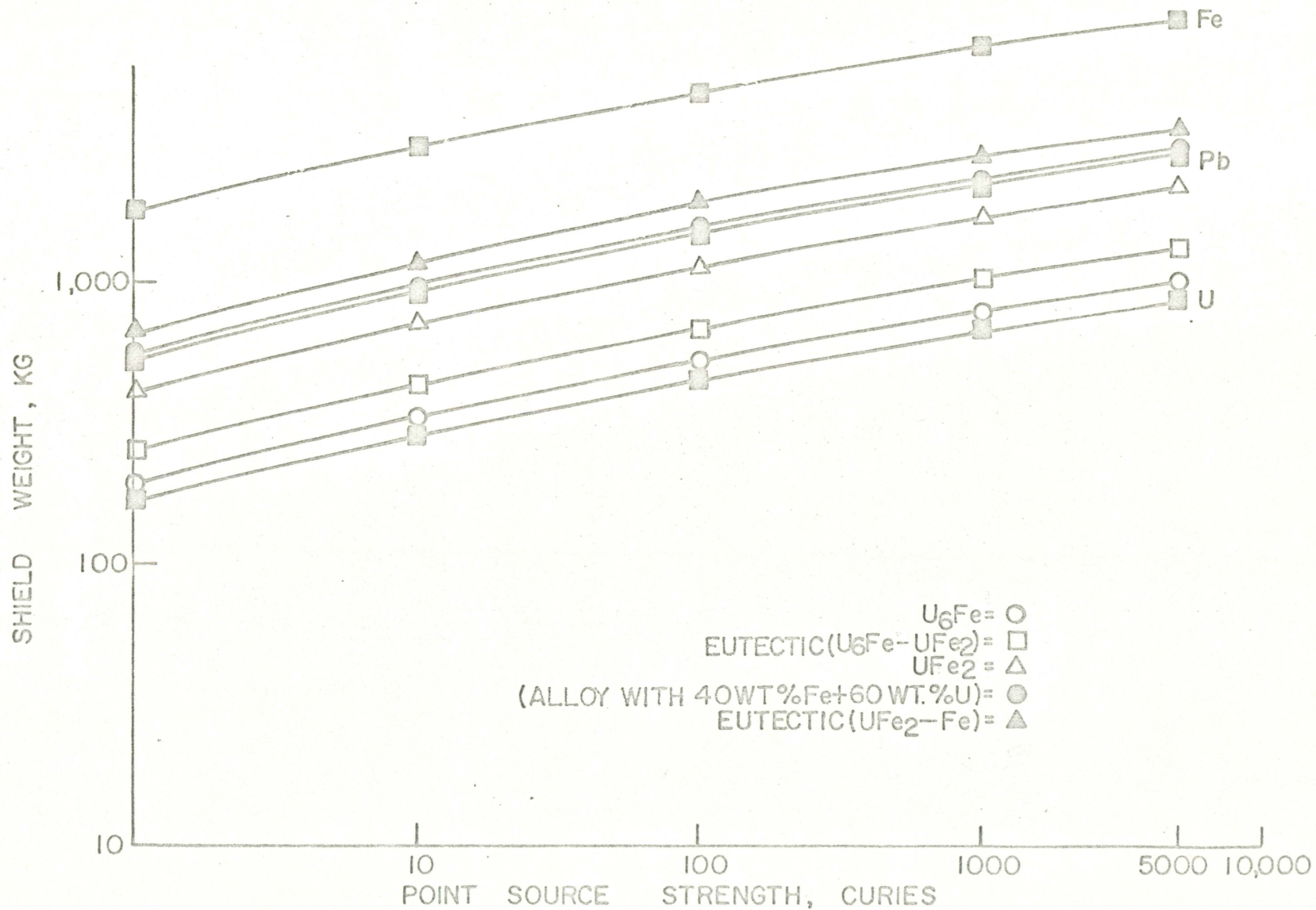


Figure 29. Shield weight requirements of selected materials for gamma rays of energy 3 Mev





## CONCLUSIONS

The shielding properties of the compounds of the uranium-iron system have certain advantages over either iron or uranium alone. Except for  $U_6Fe$  the compounds under investigation are not pyrophoric in contrast to uranium, which is pyrophoric. Some of the compounds have melting points higher than that of pure uranium. These alloys would be more reliable shields in the case of accidental fire. The results demonstrated that the use of any of the compounds under investigation in place of iron will reduce both the weight and size of the shield significantly.

The analyses that have been done in this work have been concerned only with shielding properties. There are many physical and mechanical properties yet to be determined. They must be shown to be compatible with shielding requirements before one can use any of the compositions as a shielding material.

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